Derivatives with respect to time

Derivatives with respect to time are a major application of Calculus. Here are some examples:

Quantity:	Derivative (with respect to time):	Second derivative:	Third derivative:
Position	Velocity	Acceleration	Jerk
Velocity	Acceleration	Jerk	
Speed	Colloquial acceleration		
Acceleration	Jerk		
Net wealth	Net income		
National debt	National deficit		

Position tells you where something is, while **velocity** tells you how it is moving, that is how its position is changing with time. Velocity is not quite the same thing as **speed**, since velocity keeps track of direction as well. (In this class, most problems involving motion will take place in only one dimension, so there are two directions, represented by positive and negative velocity, while speed is the absolute value of velocity.)

The derivative of velocity with respect to time, in other words the second derivative of position with respect to time, is **acceleration** in the technical sense of this term. On the other hand, the derivative of speed is **colloquial acceleration**, which reflects how the term is used in everyday life. Colloquially, we say that an object is accelerating if its speed increases with time (in other words if it is speeding up) and decelerating if its speed decreases (in other words if it is slowing down). But in the technical sense of the term, if an object is moving in the negative direction and slows down, then its velocity is becoming less negative and more positive, and so its acceleration is positive, even though its colloquial acceleration is negative. (For motion in more than one dimension, its even possible for the colloquial acceleration to be zero even while the technical acceleration is far from zero; this happens when changing direction while travelling at a constant speed.)

The time derivative of acceleration (in the technical sense) is **jerk**; that makes jerk the second derivative of velocity and the third derivative of position. Whereas position and velocity can't be directly felt, you feel acceleration as a pressure or absence thereof (a sense of falling or being held or pushed), and a sudden change in that acceleration is a jerk or yank. In engineering, acceleration must be controlled because it can destroy objects by crushing; jerk must be controlled because it can destroy objects by breaking them apart. Even higher derivatives are sometimes also studied, although the terminology varies.

Turning to finances, your **net wealth** is the total value of all assets that you own minus the value of all of your debts. (If you owe more than you own, then your net wealth is negative.) This is measured in units of money, such as dollars. Your **net income**, on the other hand, is the total value of everything that you receive (as wages, gifts, and so forth) in a period of time minus the value of your expenses. This is measured in units of money per unit of time, such as dollars per year. In finance, the default unit of time is a year, so you'll often say that someone's income is so many dollars, but this really means so many dollars *per year*. Unlike physical motion, money goes in and comes out in discrete chunks, so the continuous ideas of Calculus are only an approximation, but they can be a good approximation for some purposes.

Turning from personal finances to national, a country's government usually has some debt, called the country's **national debt**, and if the government spends more than it receives from taxes and other revenue, then the difference is the **national deficit**. The debt is the total amount of money owed by the government, while the deficit is the additional amount that has to be borrowed in a given period of time. Again, deficit should really be measured in units of money per unit of time; so if someone says the the U.S. national deficit is nearly 500 billion dollars, this really means 500 billion dollars *per year*. This is the same as 5000 billion dollars (or 5 trillion dollars) per decade (since a decade is 10 years). On the other hand, when they say that the U.S. national debt is nearly 20 trillion dollars, then they are saying exactly what they mean; this is the net result of all of the deficits (and occasional surpluses, which are negative deficits) in the past.

In 2010, there was a widely cited economics paper (Reinhart & Rogoff) that argued that a country tended towards economic disaster as its government's debt approached its GDP (gross domestic product,

a measure of a country's overall income). In 2013, a review (Herndon, Ash, & Pollin) found statistical errors that undemined the paper's conclusions, and this made the mainstream news media for a while. This should have just been the normal process of science: a flawed idea being corrected. But it was big news because Reinhart & Rogoff had struck an intuitive chord; it made sense that of course your debt should always be well below your ability to pay it off. But in fact that only sounds reasonable if you ignore the units! Reinhart & Rogoff's conclusion was really that a country was courting disaster if its government's debt was close to its GDP *times one year*; otherwise, the units of measurement don't make sense. The idea that a country should have enough income to pay off its government's debt becomes the idea that a country should have enough income to pay off its government's debt *in one year* (if all income were devoted to this purpose), and there's no intuitive reason why that should be necessary to avoid economic ruin. (It is still true that a country's economy tends to be better off when its government debt divided by its GDP is lower than otherwise, and it's conceivable that there could be some reason that there's something special about when that quotient is close to one year; but there isn't.)