8.7.9 First,

$$\int \frac{2\,\mathrm{d}x}{x^2-1} = \int \left(\frac{1}{x-1} - \frac{1}{x+1}\right) \mathrm{d}x = \ln|x-1| - \ln|x+1| + C = \ln\left|\frac{x-1}{x+1}\right| + C.$$

Then for $a \leq -2$,

$$\int_a^{-2} \frac{2\,\mathrm{d}x}{x^2-1} = \left(\ln\left|\frac{x-1}{x+1}\right|\right)\Big|_a^{-2} = \ln 3 - \ln\frac{a-1}{a+1}.$$

Finally,

$$\int_{-\infty}^{-2} \frac{2\,\mathrm{d}x}{x^2-1} = \lim_{a\to-\infty} \int_a^{-2} \frac{2\,\mathrm{d}x}{x^2-1} = \lim_{a\to-\infty}\left(\ln 3 - \ln\frac{a-1}{a+1}\right)$$
$$= \ln 3 - \ln \lim_{a\to-\infty} \frac{1-1/a}{1+1/a} = \ln 3 - \ln\frac{1-0}{1+0} = \ln 3.$$

8.7.43 First,

$$\int \frac{\mathrm{d}x}{1-x^2} = \int \left(\frac{1/2}{1-x} - \frac{1/2}{1+x}\right) \mathrm{d}x = -\frac{1}{2}\ln|1-x| - \frac{1}{2}\ln|1+x| + C = -\frac{1}{2}\ln\left|\frac{1-x}{1+x}\right| + C.$$

Then for $0 \leq b < 1$,

$$\int_0^b \frac{\mathrm{d}x}{1-x^2} = \left(-\frac{1}{2}\ln\left|\frac{1-x}{1+x}\right|\right)\Big|_0^b = -\frac{1}{2}\ln\frac{1-b}{1+b};$$

and for $1 < a \leq 2$,

$$\int_a^2 \frac{\mathrm{d}x}{1-x^2} = \left(-\frac{1}{2}\ln\left|\frac{1-x}{1+x}\right|\right)\Big|_a^2 = \frac{1}{2}\ln 3 + \frac{1}{2}\ln\frac{a-1}{a+1}.$$

Finally,

$$\int_0^2 \frac{\mathrm{d}x}{1-x^2} = \int_0^1 \frac{\mathrm{d}x}{1-x^2} + \int_1^2 \frac{\mathrm{d}x}{1-x^2} = \lim_{b\to1^-}\int_0^b \frac{\mathrm{d}x}{1-x^2} + \lim_{a\to1^+}\int_a^2 \frac{\mathrm{d}x}{1-x^2}$$
$$= \lim_{b\to1^-}\left(-\frac{1}{2}\ln\frac{1-b}{1+b}\right) + \lim_{a\to1^+}\left(\frac{1}{2}\ln 3 + \frac{1}{2}\ln\frac{a-1}{a+1}\right)$$
$$= -\frac{1}{2}\lim_{t\to0^+}\ln t + \frac{1}{2}\ln 3 + \frac{1}{2}\lim_{t\to0^+}\ln t = \infty - \infty,$$

which **diverges**.