9.3.3 The obvious function to try for the Integral Test is

$$f(x) = \frac{1}{x^2+4}.$$

This function is always defined (it would be enough if it were eventually defined), so the test might apply. The function only takes positive values (it would be enough if it eventually took only non-negative values), so the test still might apply. This function is eventually decreasing (it would be enough if it were eventually non-increasing), because

$$f'(x) = -\frac{2x}{\left(x^2+4\right)^2}$$

is negative for $x > 0$, so now the test really does apply.

The integral

$$\int_1^\infty f(x)\,\mathrm{d}x = \int_1^\infty \frac{\mathrm{d}x}{x^2+4} = \lim_{b\to\infty} \left(\frac{1}{2}\arctan\frac{x}{2}\right)\Bigg|_1^b = \frac{\pi}{4} - \frac{1}{2}\arctan\frac{1}{2}$$

converges, so the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2+4}$$

also **converges**.