9.7.5 Since

$$\frac{(x-2)^n}{10^n} = 1\left(\frac{x-2}{10}\right)^n,$$

this is a geometric series, which converges if and only if

$$\left|\frac{x-2}{10}\right| < 1,$$

in other words if and only if

$$-8 < x < 12.$$

(In that case,

$$\frac{1\left(\frac{x-2}{10}\right)^0}{1-\frac{x-2}{10}} = \frac{10}{12-x}$$

is what it converges to.)

a The radius of convergence in x is

$$\frac{(12)-(-8)}{2} = 10,$$

and the interval of convergence in x is

$$(-8, 12).$$

b I know that this, as a power series, converges absolutely at least in the interior of its interval of convergence, which in this case is all of its interval of convergence. So it converges absolutely exactly for

$$-8 < x < 12.$$

c Similarly, it converges conditionally **never**.

9.7.18 This series is *not* a geometric series, so I first apply the Root Test or the Ratio Test to the absolute values. Either test would work, but the Ratio Test is probably a little easier here.

$$\lim_{n\to\infty} \frac{\left|\frac{(n+1)x^{n+1}}{4^{n+1}\left((n+1)^2+1\right)}\right|}{\left|\frac{nx^n}{4^n(n^2+1)}\right|} = \lim_{n\to\infty} \frac{(n+1)\,|x|(n^2+1)}{4(n^2+2n+2)n} = \lim_{n\to\infty} \frac{\left(1+\frac{1}{n}\right)\left(1+\frac{1}{n^2}\right)}{4\left(1+\frac{2}{n}+\frac{2}{n^2}\right)}\,|x| = \frac{|x|}{4}.$$

So, the series converges absolutely at least if this is less than 1, in other words if

$$-4 < x < 4.$$

Now check the endpoints; if $x = 4$, then

$$\frac{nx^n}{4^n(n^2+1)} = \frac{n(4)^n}{4^n(n^2+1)} = \frac{n}{n^2+1}.$$

I can compare the sum of this to the sum of $1/n$ using the Limit Comparison Test, to conclude that it diverges. Next, if $x = -4$, then

$$\frac{nx^n}{4^n(n^2+1)} = \frac{n(-4)^n}{4^n(n^2+1)} = (-1)^n\frac{n}{n^2+1}.$$

I can use the Alternating Series Test to conclude that this converges. In fact, it converges conditionally, because the series of absolute values is the same series that I appeared when $x = 4$ above.

a The radius of convergence in x is

$$\frac{(4)-(-4)}{2}=4,$$

and the interval of convergence in x is

$$[-4,4).$$

b I know that this, as a power series, converges absolutely at least in the interior of its interval of convergence. Since it converges only conditionally for $x=-4$, it converges absolutely exactly for

$$-4<x<4.$$

c It converges conditionally exactly for

$$x=-4.$$

9.7.39 This time the Ratio Test is definitely easier.

$$\lim_{n\to\infty}\frac{\left|\frac{(n+1)!^2}{2^{n+1}(2(n+1))!}x^{n+1}\right|}{\left|\frac{n!^2}{2^n(2n)!}x^n\right|}=\lim_{n\to\infty}\frac{(n+1)^2n!^2(2n)!\,|x|}{2(2n+2)(2n+1)(2n)!n!^2}=\lim_{n\to\infty}\frac{\left(1+\frac{1}{n}\right)^2}{2\left(2+\frac{2}{n}\right)\left(2+\frac{1}{n}\right)}|x|=\frac{|x|}{8}.$$

So, the series converges absolutely at least if this is less than 1, in other words if

$$-8<x<8.$$

Thus, the radius of convergence in x is

$$\frac{(8)-(-8)}{2}=8.$$

(I have *not* done the work to determine the *interval* of convergence, which the problem didn't ask for.)