

9.9.12 Since

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1!},$$

it follows that

$$x^2 \sin x = x^2 \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1!} = \sum_{n=0}^{\infty} x^2 (-1)^n \frac{x^{2n+1}}{2n+1!} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+3}}{2n+1!}.$$

9.10.29 Since

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \sum_{n=3}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{1}{2}x^2 + \sum_{n=3}^{\infty} \frac{x^n}{n!},$$

it follows that

$$\frac{e^x - (1+x)}{x^2} = \frac{\left(1 + x + \frac{1}{2}x^2 + \sum_{n=3}^{\infty} \frac{x^n}{n!}\right) - (1+x)}{x^2} = \frac{\frac{1}{2}x^2 + \sum_{n=3}^{\infty} \frac{x^n}{n!}}{x^2} = \frac{1}{2} + \sum_{n=3}^{\infty} \frac{x^{n-2}}{n!}.$$

Since $n-2$ is positive for $n \geq 3$,

$$\lim_{x \rightarrow 0} \frac{e^x - (1+x)}{x^2} = \lim_{x \rightarrow 0} \left(\frac{1}{2} + \sum_{n=3}^{\infty} \frac{x^{n-2}}{n!} \right) = \frac{1}{2} + 0 = \frac{1}{2}.$$