6.5.9

$$W = \int_0^{180} 4.5x \, \mathrm{d}x = 4.5\left(\frac{x^2}{2}\right)\bigg|_0^{180} = 4.5\left(\frac{(180)^2}{2} - \frac{(0)^2}{2}\right) = 72\,900,$$

so the work done is 72,900 **foot-pounds**. (With units, $W = 4.5\,\mathrm{lb/ft}\left(\frac{(180\,\mathrm{ft})^2}{2}\right)$, which will calculate the correct unit of foot-pounds.)

6.5.21 The radius at height y is $\sqrt{25 - y^2}$, so the area of the cross section at that height is

$$A = \pi\left(\sqrt{25 - y^2}\right)^2 = \pi(25 - y^2).$$

Then the weight of the infinitely thin slice at that height is

$$9800A \, \mathrm{d}y = 9800\pi(25 - y^2) \, \mathrm{d}y.$$

The slice is raised to height 4, so a distance of $4 - y$. Thus, the overall work is

$$\begin{aligned} W &= \int_{-5}^{0} 9800\pi(25 - y^2)(4 - y) \, \mathrm{d}y \\ &= 9800\pi \int_{-5}^{0} (100 - 25y - 4y^2 + y^3) \, \mathrm{d}y \\ &= 9800\pi\left(100y - 25\frac{y^2}{2} - 4\frac{y^3}{3} + \frac{y^4}{4}\right)\bigg|_{-5}^{0} \\ &= -9800\pi\left(100(-5) - 25\frac{(-5)^2}{2} - 4\frac{(-5)^3}{3} + \frac{(-5)^4}{4}\right) = \frac{14\,393\,750\pi}{3}. \end{aligned}$$

Sinc the density $9800\,\mathrm{N/m^3}$ of water is only given to two significant digits, let us round this off to 15 **million joules**. (With units, $W = \int_{-5\,\mathrm{m}}^{0} 9800\,\mathrm{N/m^3}\,\pi(25\,\mathrm{m}^2 - y^2)(4\,\mathrm{m} - y)\,\mathrm{d}y$, which will calculate the correct unit of newton-metres, or joules.)