7.2.7 There's no hope of *solving* this equation with the technique that we know, but that's OK; we only have to *check* the solution that's given to us.

First, if $y = 1/x \cos x$, then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{x^2}\cos x - \frac{1}{x}\sin x,$$

so

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + y = x\left(-\frac{1}{x^2}\cos x - \frac{1}{x}\sin x\right) + \left(\frac{1}{x}\cos x\right) = -\frac{1}{x}\cos x - \sin x + \frac{1}{x}\cos x = -\sin x,$$

so the equation is satisfied.

Next, if $x = \pi/2$, then

$$y = \frac{1}{\pi/2}\cos\left(\frac{\pi}{2}\right) = 0,$$

so the initial condition is satisfied.

Therefore, the solution given is correct.

7.2.11

$$\begin{aligned}
\frac{\mathrm{d}y}{\mathrm{d}x} &= \mathrm{e}^{x-y};\\
\mathrm{d}y &= \frac{\mathrm{e}^x}{\mathrm{e}^y}\mathrm{d}x;\\
\mathrm{e}^y\,\mathrm{d}y &= \mathrm{e}^x\,\mathrm{d}x;\\
\int \mathrm{e}^y\,\mathrm{d}y &= \int \mathrm{e}^x\,\mathrm{d}x;\\
\mathrm{e}^y &= \mathrm{e}^x + C;\\
y &= \ln(\mathrm{e}^x + C).
\end{aligned}$$

7.2.35 The easiest way to do this is to use

$$A = P2^{-t/h}$$

with $h = 24\,360$ years. Solve for t:

$$\begin{aligned}
A &= P2^{-t/h};\\
\frac{A}{P} &= 2^{-t/h};\\
\log_2\left(\frac{A}{P}\right) &= -\frac{t}{h};\\
t &= -h\log_2\left(\frac{A}{P}\right).
\end{aligned}$$

When 80 percent is gone, 20 percent is left, so $A = 20/100\,P$. Therefore,

$$t = -24\,360\,\mathrm{y}\log_2\left(\frac{20}{100}\right) = -24\,360\frac{\ln 0.2}{\ln 2}\,\mathrm{y} \approx 56\,600\,\mathrm{y}$$

(where ‘y’ is the abbreviation for years).