

8.2.11 Either

$$\int \sin^3 x \cos^3 x \, dx = \int \sin^3 x (1 - \sin^2 x) \cos x \, dx = \int (\sin^3 x - \sin^5 x) \, d(\sin x) = \frac{1}{4} \sin^4 x - \frac{1}{6} \sin^6 x + C$$

or

$$\int \sin^3 x \cos^3 x \, dx = -\int (1 - \cos^2 x) \cos^3 x (-\sin x \, dx) = \int (\cos^5 x - \cos^3 x) \, d(\cos x) = \frac{1}{6} \cos^6 x - \frac{1}{4} \cos^4 x + C.$$

(The constant in the second answer is $1/12$ more than the constant in the first answer.)

8.2.25 First,

$$\int_0^\pi \sqrt{1 - \sin^2 t} \, dt = \int_0^\pi \sqrt{\cos^2 t} \, dt = \int_0^\pi |\cos t| \, dt.$$

For $0 \leq t \leq \pi/2$, we have $\cos t \geq 0$; for $\pi/2 \leq t \leq \pi$, we have $\cos t \leq 0$. Thus,

$$\begin{aligned} \int_0^\pi |\cos t| \, dt &= \int_0^{\pi/2} \cos t \, dt - \int_{\pi/2}^\pi \cos t \, dt \\ &= (\sin t) \Big|_0^{\pi/2} - (\sin t) \Big|_{\pi/2}^\pi = \sin\left(\frac{\pi}{2}\right) - \sin 0 - \sin \pi + \sin\left(\frac{\pi}{2}\right) = 2. \end{aligned}$$

8.2.41

$$\int \sec^4 \theta \, d\theta = \int (\tan^2 \theta + 1) \sec^2 \theta \, d\theta = \int (\tan^2 \theta + 1) \, d(\tan \theta) = \frac{1}{3} \tan^3 \theta + \tan \theta + C.$$