8.3.11 I want $y = 7\sec\theta$, so that

$$y^2 - 49 = (7\sec\theta)^2 - 49 = 49\sec^2\theta - 49 = 49(\sec^2\theta - 1) = 49\tan^2\theta = (7\tan\theta)^2.$$

In other words, I want $\cos\theta = 7/y$, so let θ be $\arccos(7/y)$. Since $y > 7$ is positive, so is $7/y > 0$, so θ is an acute angle, so $\tan\theta$ is positive. Therefore,

$$\sqrt{y^2 - 49} = \tan\theta$$

(rather than $|\tan\theta|$ in general). I also have

$$\mathrm{d}y = 7\sec\theta\tan\theta\,\mathrm{d}\theta.$$

Now,

$$\begin{aligned}\int \frac{\sqrt{y^2-49}}{y}\,\mathrm{d}y &= \int \frac{\tan\theta}{\sec\theta} 7\sec\theta\tan\theta\,\mathrm{d}\theta = 7\int \tan^2\theta\,\mathrm{d}\theta = 7\int(\sec^2\theta - 1)\,\mathrm{d}\theta \\ &= 7(\tan\theta - \theta) + C = 7\sqrt{y^2-49} - 7\arccos\frac{7}{y} + C.\end{aligned}$$