

8.4.7 Since $t^2 - 5t + 6 = (t - 2)(t - 3)$ and $t^2 + 8$ is the same degree, somehow

$$\frac{t^2 + 8}{t^2 - 5t + 6} = A + \frac{B}{t - 2} + \frac{C}{t - 3}.$$

Multiplying by the common denominator,

$$t^2 + 8 = At^2 - 5At + 6A + Bt - 3B + Ct - 2C.$$

Gathering like terms,

$$\begin{aligned} A &= 1, \\ -5A + B + C &= 0, \text{ and} \\ 6A - 3B - 2C &= 8; \end{aligned}$$

so $A = 1$, $B = -12$, and $C = 17$. Therefore,

$$\frac{t^2 + 8}{t^2 - 5t + 6} = 1 - \frac{12}{t - 2} + \frac{17}{t - 3}.$$

(It follows that

$$\int \frac{t^2 + 8}{t^2 - 5t + 6} dt = t - 12 \ln |t - 2| + 17 \ln |t - 3| + C,$$

although this wasn't asked for.)