8.6.15.a To estimate

$$\int_a^b f(t)\,\mathrm{d}t = \int_0^2 (t^3 + t)\,\mathrm{d}t,$$

we have

$$\begin{aligned} f(t) &= t^3 + t, \text{ so} \\ f'(t) &= 3t^2 + 1, \text{ so} \\ f''(t) &= 6t. \end{aligned}$$

Since f'' is increasing (say because $f'''(t) = 6$ is positive), the maximum value M of f'' on $[0, 2]$ is $f''(2) = 6(2) = 12$. With n steps, we have

$$|E| \leq \frac{M(b-a)^3}{12n^2} = \frac{12(2-0)^3}{12n^2} = \frac{8}{n^2}.$$

To ensure $|E| < 10^{-4}$, then,

$$\begin{aligned} 10^{-4} &> \frac{8}{n^2}, \text{ so} \\ n^2 &> \frac{8}{10^{-4}} = 80\,000, \text{ so} \\ n &> \sqrt{80\,000} \approx 282.8. \end{aligned}$$

Since n must be a natural number,

$$n \geq 283.$$