

1 Given

$$\frac{dy}{dx} = e^{x-y}$$

and $y = 1$ when $x = 0$, find y as a function of x .

a $y = \ln(e^x - 1 + e)$

b $y = \ln(e^{x+1})$

c $y = \ln(e^x - 1 - e^2)$

d $y = \ln(e^{x+1} + e)$

2 A tank has the shape of a cylinder, with a height of 8 metres and a circular base with a radius of 5 metres. The tank is half full of a liquid which weighs ρ newtons per cubic metre. How much work is required to pump the water 3 metres above the top of the tank?

a $900\pi\rho$ joules

b $600\pi\rho$ joules

c $500\pi\rho$ joules

d $800\pi\rho$ joules

3 Integrate

$$\int_1^e x \ln x \, dx.$$

a $\frac{3}{4}e^2 - \frac{1}{2}$

b $\frac{1}{4} - \frac{1}{4}e^2$

c $\frac{3}{4}e^2 - \frac{1}{4}$

d $\frac{1}{4} + \frac{1}{4}e^2$

4 Approximate

$$\int_0^1 \frac{dx}{2-x}$$

using 4 steps and the smallest value in each interval.

a $\frac{743}{840} \approx 0.8845$

b $\frac{533}{840} \approx 0.6345$

c $\frac{743}{210} \approx 3.5381$

d $\frac{533}{210} \approx 2.5381$

5 Integrate

$$\int_2^{\infty} \frac{dx}{x^2}.$$

a $\frac{1}{2}$

b $-\frac{1}{2}$

c ∞ (or undefined)

d 2

6 Find the sum

$$\sum_{n=2}^{\infty} \frac{(-2)^{n+1}}{3^n}.$$

a $\frac{8}{3}$

b $-\frac{8}{15}$

c $\frac{4}{15}$

d $-\frac{6}{5}$

7 Which of the following tests will determine the convergence of

$$\sum_{n=1}^{\infty} \frac{3n-1}{n^2}?$$

a The Limit Comparison Test, comparing to $\sum_{n=1}^{\infty} \frac{1}{n}$

b The Direct Comparison Test, comparing to $\sum_{n=1}^{\infty} \frac{3}{n}$

c Both of the above

d None of the above

8 Which of the following tests will determine the convergence of

$$\sum_{n=1}^{\infty} \frac{4^n}{5^n \sqrt{n}}?$$

a The Root Test

b The Ratio Test

c Both of the above

d None of the above

9 Does

$$\sum_{n=1}^{\infty} (-1)^n \frac{5}{n+1}$$

converge conditionally or absolutely?

- a Absolutely
- b Conditionally
- c Both of the above
- d None of the above

10 What is the interval of convergence (in x) of

$$\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{2^n n}$$

- a $[-4, 0)$
- b $(-4, 0)$
- c $(-4, 0]$
- d $[-4, 0]$

11 What is the Taylor series of

$$f(x) = \frac{1}{x+2}$$

at $x = 3$?

- a $f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{(x-3)^n}{5^{n+1} n!}$
- b $f(x) = \sum_{n=1}^{\infty} (-1)^n \frac{(x-3)^n}{5^n n!}$
- c $f(x) = \sum_{n=1}^{\infty} (-1)^n \frac{(x-3)^n}{5^n}$
- d $f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{(x-3)^n}{5^{n+1}}$

12 Write

$$r = \cot \theta \csc \theta$$

in rectangular (Cartesian) coordinates.

- a $y = x^2 \sqrt{x^2 + y^2}$
- b $x = y^2 \sqrt{x^2 + y^2}$
- c $y = x^2$
- d $x = y^2$

13 Given

$$r = 1 + \sin \theta$$

in polar coordinates, what is the slope of the tangent line when $\theta = \pi/3$?

a Undefined (or infinite)

b 1

c -1

d 0

14 Given

$$r = 3 \sin \theta$$

in polar coordinates, what is the length of the curve from $\theta = 0$ to $\theta = \pi/4$?

a 3π

b $\frac{3\pi}{8}$

c $\frac{3\pi}{2}$

d $\frac{3\pi}{4}$

15 What is the magnitude (length) of $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k} = \langle -2, 2, -1 \rangle$?

a 3

b 1

c 5

d -1

16 Given $\mathbf{u} = 3\mathbf{i} + 4\mathbf{k} = \langle 3, 0, 4 \rangle$ and $\mathbf{v} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} = \langle -2, 2, -1 \rangle$, what is the projection of \mathbf{v} onto the direction of \mathbf{u} ?

a $-\frac{6}{5}\mathbf{i} - \frac{8}{5}\mathbf{k} = \langle -\frac{6}{5}, 0, -\frac{8}{5} \rangle$

b $20\mathbf{i} - 20\mathbf{j} + 10\mathbf{k} = \langle 20, -20, 10 \rangle$

c $\frac{20}{9}\mathbf{i} - \frac{20}{9}\mathbf{j} + \frac{10}{9}\mathbf{k} = \langle \frac{20}{9}, -\frac{20}{9}, \frac{10}{9} \rangle$

d $-30\mathbf{i} - 40\mathbf{k} = \langle -30, 0, -40 \rangle$

17 Given $\mathbf{u} = 3\mathbf{i} - 4\mathbf{k} = \langle 3, 0, -4 \rangle$ and $\mathbf{v} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} = \langle -2, 2, -1 \rangle$, what is the cross product $\mathbf{u} \times \mathbf{v}$?

a $-8\mathbf{i} + 5\mathbf{j} + 6\mathbf{k} = \langle -8, 5, 6 \rangle$

b $8\mathbf{i} + 11\mathbf{j} + 6\mathbf{k} = \langle 8, 11, 6 \rangle$

c $4\mathbf{i} - 2\mathbf{j} - 6\mathbf{k} = \langle 4, -2, -6 \rangle$

d $-6\mathbf{i} + 4\mathbf{k} = \langle -6, 0, 4 \rangle$

Answers

1 A, 2 A, 3 D, 4 B, 5 A, 6 B, 7 A, 8 C, 9 B, 10 C, 11 D, 12 D, 13 C, 14 D, 15 A, 16 A, 17 B