1 Given

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{x-y}$$

and $y = 1$ when $x = 0$, find y as a function of x.

a $\;y = \ln\left(\mathrm{e}^x - 1 + \mathrm{e}\right)$

b $\;y = \ln\left(\mathrm{e}^{x+1}\right)$

c $\;y = \ln\left(\mathrm{e}^x - 1 - \mathrm{e}^2\right)$

d $\;y = \ln\left(\mathrm{e}^{x+1} + \mathrm{e}\right)$

2 A tank has the shape of a cylinder, with a height of 8 metres and a circular base with a radius of 5 metres. The tank is half full of a liquid which weighs 10 newtons per cubic metre. How much work is required to pump the water 3 metres above the top of the tank?

a $\;9000\pi$ joules

b $\;6000\pi$ joules

c $\;5000\pi$ joules

d $\;8000\pi$ joules

3 Integrate

$$\int_1^{\mathrm{e}} x \ln x \,\mathrm{d}x.$$

a $\;\dfrac{3}{4}\mathrm{e}^2 - \dfrac{1}{2}$

b $\;\dfrac{1}{4} - \dfrac{1}{4}\mathrm{e}^2$

c $\;\dfrac{3}{4}\mathrm{e}^2 - \dfrac{1}{4}$

d $\;\dfrac{1}{4} + \dfrac{1}{4}\mathrm{e}^2$

4 Approximate

$$\int_0^1 \frac{\mathrm{d}x}{2 - x}$$

using the Trapezoid Rule with 4 trapezoids.

a $\;\dfrac{1171}{840} \approx 1.3940$

b $\;\dfrac{1171}{1680} \approx 0.6970$

c $\;\dfrac{743}{840} \approx 0.8845$

d $\;\dfrac{743}{1050} \approx 0.7076$

5 Integrate

$$\int_2^\infty \frac{\mathrm{d}x}{x^2}.$$

a $\dfrac{1}{2}$

b $-\dfrac{1}{2}$

c ∞ (or undefined)

d 2

6 Find the sum

$$\sum_{n=2}^\infty \frac{(-2)^{n+1}}{3^n}.$$

a $\dfrac{8}{3}$

b $-\dfrac{8}{15}$

c $\dfrac{4}{15}$

d $-\dfrac{6}{5}$

7 Which of the following tests will determine the convergence of

$$\sum_{n=1}^\infty \frac{3n-1}{n^2}?$$

a The Limit Comparison Test, comparing to $\displaystyle\sum_{n=1}^\infty \frac{1}{n}$

b The Direct Comparison Test, comparing to $\displaystyle\sum_{n=1}^\infty \frac{3}{n}$

c Both of the above

d Neither of the above

8 Which of the following tests will determine the convergence of

$$\sum_{n=1}^\infty \frac{4^n}{5^n \sqrt{n}}?$$

a The Root Test

b The Ratio Test

c Both of the above

d Neither of the above

9 Does

$$\sum_{n=1}^{\infty} (-1)^n \frac{5}{n+1}$$

converge conditionally or absolutely?

a Absolutely

b Conditionally

c Both of the above

d Neither of the above

10 What is the interval of convergence (in x) of

$$\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{2^n n}?$$

a $[-4,0)$

b $(-4,0)$

c $(-4,0]$

d $[-4,0]$

11 What is the Taylor series of

$$f(x) = \frac{1}{x+2}$$

at $x = 3$?

a $f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{(x-3)^n}{5^{n+1} \, n!}$

b $f(x) = \sum_{n=1}^{\infty} (-1)^n \frac{(x-3)^n}{5^n \, n!}$

c $f(x) = \sum_{n=1}^{\infty} (-1)^n \frac{(x-3)^n}{5^n}$

d $f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{(x-3)^n}{5^{n+1}}$

12 Write

$$r = \cot\theta \csc\theta$$

in rectangular (Cartesian) coordinates.

a $y = x^2 \sqrt{x^2 + y^2}$

b $x = y^2 \sqrt{x^2 + y^2}$

c $y = x^2$

d $x = y^2$

13 Given
$$r = 1 + \sin\theta$$
in polar coordinates, what is the slope of the tangent line when $\theta = \pi/3$?

a Undefined (or infinite)

b 1

c -1

d 0

14 Given
$$r = 3\sin\theta$$
in polar coordinates, what is the length of the curve from $\theta = 0$ to $\theta = \pi/4$?

a 3π

b $\dfrac{3\pi}{8}$

c $\dfrac{3\pi}{2}$

d $\dfrac{3\pi}{4}$

Answers

1 A, 2 A, 3 D, 4 B, 5 A, 6 B, 7 A, 8 C, 9 B, 10 C, 11 D, 12 D, 13 C, 14 D