

- 5.45** This one is simpler if you do the substitution only in the cosine term and do the polynomial terms directly. It's very tempting to substitute $u = 2\theta + 1$ in everything, but that makes it more complicated and easier to mess up. However, if you want to do that anyway, here's how it would go:

First,

$$\begin{aligned}u &= 2\theta + 1, \\du &= 2 d\theta, \\d\theta &= \frac{1}{2} du;\end{aligned}$$

then

$$\begin{aligned}\int (2\theta + 1 + 2 \cos(2\theta + 1)) d\theta &= \int (u + 2 \cos u) \left(\frac{1}{2} du\right) = \int \left(\frac{1}{2}u + \cos u\right) du \\&= \frac{1}{4}u^2 + \sin u + C = \frac{1}{4}(2\theta + 1)^2 + \sin(2\theta + 1) + C.\end{aligned}$$

You can simplify this expression:

$$\frac{1}{4}(2\theta + 1)^2 + \sin(2\theta + 1) + C = \frac{1}{4}(4\theta^2 + 4\theta + 1) + \sin(2\theta + 1) + C = \theta^2 + \theta + \frac{1}{4} + \sin(2\theta + 1) + C.$$

The answer in the back of the book (which you'll get if you don't substitute in the polynomial terms) is

$$\theta^2 + \theta + \sin(2\theta + 1) + C;$$

these are equivalent, since the extra $1/4$ may be absorbed into the constant C .