Practice Exam   Math-1700-es31   **2015 March 11**

There will be a comprehensive final exam taken in class on March 18 Wednesday. The exam will be multiple choice, with no partial credit (except possibly on extra credit problems).

For the exam, you may use one sheet of notes that you wrote yourself. However, you may not use your book or anything else not written by you. You certainly should not talk to other people! Calculators are allowed, although you shouldn't really need them, but not communication devices (like cell phones).

1 Given
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{x-y}$$
and $y = 1$ when $x = 0$, find y as a function of x.

a $y = \ln\left(\mathrm{e}^x - 1 + \mathrm{e}\right)$

b $y = \ln\left(\mathrm{e}^{x+1}\right)$

c $y = \ln\left(\mathrm{e}^x - 1 - \mathrm{e}^2\right)$

d $y = \ln\left(\mathrm{e}^{x+1} + \mathrm{e}\right)$

2 A tank has the shape of a cylinder, with a height of 8 metres and a circular base with a radius of 5 metres. The tank is half full of a liquid which weighs 10 newtons per cubic metre. How much work is required to pump the water 3 metres above the top of the tank?

a 9000π joules

b 6000π joules

c 5000π joules

d 8000π joules

3 Integrate
$$\int_1^{\mathrm{e}} x \ln x \, \mathrm{d}x.$$

a $\dfrac{3}{4}\mathrm{e}^2 - \dfrac{1}{2}$

b $\dfrac{1}{4} - \dfrac{1}{4}\mathrm{e}^2$

c $\dfrac{3}{4}\mathrm{e}^2 - \dfrac{1}{4}$

d $\dfrac{1}{4} + \dfrac{1}{4}\mathrm{e}^2$

4 Approximate
$$\int_0^1 \frac{\mathrm{d}x}{2 - x}$$
using the Trapezoid Rule with 4 trapezoids.

a $\dfrac{1171}{840} \approx 1.3940$

b $\dfrac{1171}{1680} \approx 0.6970$

c $\dfrac{743}{840} \approx 0.8845$

d $\dfrac{743}{1050} \approx 0.7076$

5 Integrate
$$\int_2^\infty \frac{\mathrm{d}x}{x^2}.$$

a $\dfrac{1}{2}$

b $-\dfrac{1}{2}$

c ∞ (or undefined)

d 2

6 Find the sum
$$\sum_{n=2}^\infty \frac{(-2)^{n+1}}{3^n}.$$

a $\dfrac{8}{3}$

b $-\dfrac{8}{15}$

c $\dfrac{4}{15}$

d $-\dfrac{6}{5}$

7 Which of the following tests will determine the convergence of
$$\sum_{n=1}^\infty \frac{3n-1}{n^2}?$$

a The Limit Comparison Test, comparing to $\displaystyle\sum_{n=1}^\infty \frac{1}{n}$

b The Direct Comparison Test, comparing to $\displaystyle\sum_{n=1}^\infty \frac{3}{n}$

c Both of the above

d Neither of the above

8 Which of the following tests will determine the convergence of
$$\sum_{n=1}^\infty \frac{4^n}{5^n\sqrt{n}}?$$

a The Root Test

b The Ratio Test

c Both of the above

d Neither of the above

9 Does

$$\sum_{n=1}^{\infty} (-1)^n \frac{5}{n+1}$$

converge conditionally or absolutely?

a Absolutely

b Conditionally

c Both of the above

d Neither of the above

10 What is the interval of convergence (in x) of

$$\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{2^n n}?$$

a $[-4, 0)$

b $(-4, 0)$

c $(-4, 0]$

d $[-4, 0]$

11 What is the Taylor series of

$$f(x) = \frac{1}{x+2}$$

at $x = 3$?

a $f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{(x-3)^n}{5^{n+1} \, n!}$

b $f(x) = \sum_{n=1}^{\infty} (-1)^n \frac{(x-3)^n}{5^n \, n!}$

c $f(x) = \sum_{n=1}^{\infty} (-1)^n \frac{(x-3)^n}{5^n}$

d $f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{(x-3)^n}{5^{n+1}}$

12 Write

$$r = \cot \theta \csc \theta$$

in rectangular (Cartesian) coordinates.

a $y = x^2 \sqrt{x^2 + y^2}$

b $x = y^2 \sqrt{x^2 + y^2}$

c $y = x^2$

d $x = y^2$

13 Given
$$r = 1 + \sin\theta$$
in polar coordinates, what is the slope of the tangent line when $\theta = \pi/3$?

a Undefined (or infinite)

b 1

c −1

d 0

14 Given
$$r = 3\sin\theta$$
in polar coordinates, what is the length of the curve from $\theta = 0$ to $\theta = \pi/4$?

a 3π

b $\dfrac{3\pi}{8}$

c $\dfrac{3\pi}{2}$

d $\dfrac{3\pi}{4}$

15 What is the magnitude (length) of $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k} = \langle -2, 2, -1 \rangle$?

a 3

b 1

c 5

d −1

16 Given $\mathbf{u} = 3\mathbf{i} + 4\mathbf{k} = \langle 3, 0, 4 \rangle$ and $\mathbf{v} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} = \langle -2, 2, -1 \rangle$, what is the projection of $\mathbf{v}$ onto the direction of $\mathbf{u}$?

a $-\dfrac{6}{5}\mathbf{i} - \dfrac{8}{5}\mathbf{k} = \left\langle -\dfrac{6}{5}, 0, -\dfrac{8}{5} \right\rangle$

b $20\mathbf{i} - 20\mathbf{j} + 10\mathbf{k} = \langle 20, -20, 10 \rangle$

c $\dfrac{20}{9}\mathbf{i} - \dfrac{20}{9}\mathbf{j} + \dfrac{10}{9}\mathbf{k} = \left\langle \dfrac{20}{9}, -\dfrac{20}{9}, \dfrac{10}{9} \right\rangle$

d $-30\mathbf{i} - 40\mathbf{k} = \langle -30, 0, -40 \rangle$

17 Given $\mathbf{u} = 3\mathbf{i} - 4\mathbf{k} = \langle 3, 0, -4 \rangle$ and $\mathbf{v} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} = \langle -2, 2, -1 \rangle$, what is the cross product $\mathbf{u} \times \mathbf{v}$?

a $-8\mathbf{i} + 5\mathbf{j} + 6\mathbf{k} = \langle -8, 5, 6 \rangle$

b $8\mathbf{i} + 11\mathbf{j} + 6\mathbf{k} = \langle 8, 11, 6 \rangle$

c $4\mathbf{i} - 2\mathbf{j} - 6\mathbf{k} = \langle 4, -2, -6 \rangle$

d $-6\mathbf{i} + 4\mathbf{k} = \langle -6, 0, 4 \rangle$

Answers

1 A, 2 A, 3 D, 4 B, 5 A, 6 B, 7 A, 8 C, 9 B, 10 C, 11 D, 12 D, 13 C, 14 D, 15 A, 16 A, 17 B