There will be a comprehensive final exam taken in class on March 17 Thursday. The exam will be multiple choice, with no partial credit (except possibly on extra credit problems).

For the exam, you may use one sheet of notes that you wrote yourself. However, you may not use your book or anything else not written by you. You certainly should not talk to other people! Calculators are allowed, although you shouldn't really need them, but not communication devices (like cell phones).

1 A tank has the shape of a prism, with a height of 12 feet and a square base with a width of 4 feet. The tank is two-thirds full of a liquid which weighs 10 pounds per cubic foot. How much work is required to pump the liquid 5 feet above the top of the tank?

a 4480 foot-pounds

b 11 520 foot-pounds

c 16 640 foot-pounds

d 9600 foot-pounds

2 Given

$$\frac{\mathrm{d}y}{\mathrm{d}x} = xy^3$$

and $y = -1$ when $x = 0$, find y as a function of x.

a $y = -\dfrac{1}{\sqrt{1-x^2}}$

b $y = \dfrac{1}{\sqrt{x^2-1}}$

c $y = -\dfrac{1}{\sqrt{x^2-1}}$

d $y = \dfrac{1}{\sqrt{1-x^2}}$

3 Integrate

$$\int_0^\pi x \sin x \,\mathrm{d}x.$$

a $\dfrac{1}{2}\pi^2$

b π

c $-\dfrac{1}{2}\pi^2$

d $-\pi$

4 Approximate

$$\int_2^3 \frac{\mathrm{d}x}{1-x}$$

using the Trapezoid Rule and 4 equally spaced intervals.

a $\dfrac{1498\,711}{2162\,160} \approx 0.693$

b $-\dfrac{1171}{1680} \approx -0.697$

c $\dfrac{1171}{1680} \approx 0.697$

d $-\dfrac{1498\,711}{2162\,160} \approx -0.693$

5 Integrate

$$\int_1^\infty \frac{\mathrm{d}x}{x^3}.$$

a $-\dfrac{1}{2}$

b 1

c $\dfrac{1}{2}$

d -1

6 Find the sum

$$\sum_{n=3}^{\infty} \left(\frac{n-1}{n} - \frac{n}{n+1} \right).$$

a $-\dfrac{1}{3}$

b $\dfrac{2}{3}$

c $\dfrac{3}{4}$

d $-\dfrac{1}{4}$

7 Which of the following tests will determine the convergence of

$$\sum_{n=1}^{\infty} \frac{3n-1}{n^2}?$$

a Neither of these comparison tests

b The Limit Comparison Test, comparing to $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

c Both of these comparison tests

d The Direct Comparison Test, comparing to $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n}$

8 Which of the following tests will determine the convergence of

$$\sum_{n=1}^{\infty} \frac{4^n}{5^n \sqrt{n}}?$$

a Neither of these tests

b Both of these tests

c The Ratio Test

d The Root Test

9 What is the convergence or diverence of

$$\sum_{n=1}^{\infty} (-1)^n \frac{3}{n^2+1}?$$

a It converges absolutely.

b None of the other answers is correct.

c It converges conditionally.

d It diverges.

10 What is the interval of convergence (in x) of

$$\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{2^n n}?$$

a $[-4, 0)$

b $(-4, 0)$

c $(-4, 0]$

d $[-4, 0]$

11 What is the Taylor series of

$$f(x) = \mathrm{e}^{2x}$$

at $x = 0$?

a $f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{2x^n}{n!}$

b $f(x) = \sum_{n=0}^{\infty} \frac{2x^n}{n!}$

c $f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{2^n x^n}{n!}$

d $f(x) = \sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$

12 Write

$$r^2 = \sin\theta\cos\theta$$

in rectangular (cartesian) coordinates.

a $1 = xy$

b $x^2 + y^2 = xy$

c $\left(x^2 + y^2\right)^2 = xy$

d $\left(x^2 + y^2\right)^4 = xy$

13 Given

$$r = 1 + \sin\theta$$

in polar coordinates, what is the slope of the tangent line when $\theta = \pi/3$?

a -1

b Undefined (or infinite)

c 0

d 1

14 Given

$$r = 2\cos\theta$$

in polar coordinates, what is the length of the curve from $\theta = 0$ to $\theta = \pi/4$?

a $\dfrac{\pi}{4}$

b $\dfrac{\pi}{2}$

c π

d 2π

15 What is the magnitude (length) of $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k} = \langle -2, 2, -1\rangle$?

a 5

b 1

c -1

d 3

16 Given $\mathbf{u} = 3\mathbf{i} + 4\mathbf{k} = \langle 3, 0, 4\rangle$ and $\mathbf{v} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} = \langle -2, 2, -1\rangle$, what is $2\mathbf{u} - 3\mathbf{v}$?

a $12\mathbf{i} - 6\mathbf{j} + 11\mathbf{k} = \langle 12, -6, 11\rangle$

b $12\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} = \langle 12, 2, 3\rangle$

c $12\mathbf{i} + \mathbf{k} = \langle 12, 0, 1\rangle$

d $12\mathbf{i} - 2\mathbf{j} + 5\mathbf{k} = \langle 12, -2, 5\rangle$

17 Given $\mathbf{u} = 3\mathbf{i} + 4\mathbf{k} = \langle 3, 0, 4\rangle$ and $\mathbf{v} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} = \langle -2, 2, -1\rangle$, what is the projection of $\mathbf{v}$ onto the direction of $\mathbf{u}$?

a $\frac{20}{9}\mathbf{i} - \frac{20}{9}\mathbf{j} + \frac{10}{9}\mathbf{k} = \left\langle \frac{20}{9}, -\frac{20}{9}, \frac{10}{9} \right\rangle$

b $20\mathbf{i} - 20\mathbf{j} + 10\mathbf{k} = \langle 20, -20, 10\rangle$

c $-\frac{6}{5}\mathbf{i} - \frac{8}{5}\mathbf{k} = \left\langle -\frac{6}{5}, 0, -\frac{8}{5} \right\rangle$

d $-30\mathbf{i} - 40\mathbf{k} = \langle -30, 0, -40\rangle$

18 Given $\mathbf{u} = 3\mathbf{i} - 4\mathbf{k} = \langle 3, 0, -4\rangle$ and $\mathbf{v} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} = \langle -2, 2, -1\rangle$, what is the cross product $\mathbf{u} \times \mathbf{v}$?

a $4\mathbf{i} - 2\mathbf{j} - 6\mathbf{k} = \langle 4, -2, -6\rangle$

b $8\mathbf{i} + 11\mathbf{j} + 6\mathbf{k} = \langle 8, 11, 6\rangle$

c $-6\mathbf{i} + 4\mathbf{k} = \langle -6, 0, 4\rangle$

d $-8\mathbf{i} + 5\mathbf{j} + 6\mathbf{k} = \langle -8, 5, 6\rangle$

Answers

1 C, 2 A, 3 B, 4 B, 5 C, 6 A, 7 B, 8 B, 9 A, 10 C, 11 D, 12 C, 13 A, 14 B, 15 D, 16 A, 17 C, 18 B.