14.2.23 Holding y constant,

$$\int_{x=0}^{x=y^2} 3y^3 \mathrm{e}^{xy}\,\mathrm{d}x = 3y^3 \int_{x=0}^{x=y^2} \mathrm{e}^{xy}\,\frac{\mathrm{d}(xy)}{y} = \frac{3y^3}{y}\int_{x=0}^{x=y^2} \mathrm{d}(\mathrm{e}^{xy})$$

$$= 3y^2(\mathrm{e}^{xy})\big|_{x=0}^{x=y^2} = 3y^2\Big(\mathrm{e}^{(y^2)y} - \mathrm{e}^{(0)y}\Big) = 3y^2\Big(\mathrm{e}^{y^3} - 1\Big).$$

Next,

$$\int_{y=0}^{y=1} 3y^2\Big(\mathrm{e}^{y^3} - 1\Big)\,\mathrm{d}y = \int_{y=0}^{y=1}\Big(\mathrm{e}^{y^3} - 1\Big)\,\mathrm{d}(y^3) = \int_{y=0}^{y=1}\mathrm{d}\Big(\mathrm{e}^{y^3} - y^3\Big)$$

$$= \Big(\mathrm{e}^{y^3} - y^3\Big)\Big|_{y=0}^{y=1} = \Big(\mathrm{e}^{1^3} - 1^3\Big) - \Big(\mathrm{e}^{0^3} - 0^3\Big) = \mathrm{e} - 2.$$

Thus,

$$\int_0^1\int_0^{y^2} 3y^3\mathrm{e}^{xy}\,\mathrm{d}x\,\mathrm{d}y = \int_0^1 3y^2\Big(\mathrm{e}^{y^3} - 1\Big)\,\mathrm{d}y = \mathrm{e} - 2.$$