

12.1.4 First,

$$\mathbf{r}'(t) = \langle -2 \sin 2t, 6 \cos 2t \rangle,$$

so

$$\mathbf{r}'(0) = \langle 0, 6 \rangle;$$

next,

$$\mathbf{r}''(t) = \langle -4 \cos 2t, -12 \sin 2t \rangle,$$

so

$$\mathbf{r}''(0) = \langle -4, 0 \rangle.$$

For an equation involving only x and y , don't try to solve for t (which is too complicated), but use a trigonometric identity:

$$x = \cos 2t, y = 3 \sin 2t;$$

$$\cos 2t = x, \sin 2t = \frac{y}{3};$$

$$\cos^2 2t = x^2, \sin^2 2t = \frac{y^2}{9};$$

$$x^2 + \frac{y^2}{9} = 1.$$

12.2.12 First,

$$\mathbf{r} = \int \langle 180t, 180t - 16t^2 \rangle dt = \left\langle 90t^2, 90t^2 - \frac{16}{3}t^3 \right\rangle + \mathbf{C};$$

when $t = 0$,

$$\langle 0, 100 \rangle = \mathbf{r} = \langle 0, 0 \rangle + \mathbf{C},$$

so $\mathbf{C} = \langle 0, 100 \rangle$, so

$$\mathbf{r} = \left\langle 90t^2, 90t^2 - \frac{16}{3}t^3 + 100 \right\rangle.$$

12.3.1 First

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \langle -2 \sin t, 2 \cos t, \sqrt{5} \rangle;$$

then

$$\|\mathbf{v}\| = \sqrt{4 \sin^2 t + 4 \cos^2 t + 5} = 3.$$

Thus, the unit tangent vector is

$$\mathbf{T} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \left\langle -\frac{2}{3} \sin t, \frac{2}{3} \cos t, \frac{1}{3} \sqrt{5} \right\rangle,$$

and the arclength from $t = 0$ to $t = \pi$ is

$$L = \int_0^\pi 3 dt = 3\pi.$$