

12.4.3

$$\begin{aligned} \mathbf{r} &= \langle 2t + 3, 5 - t^2 \rangle; \\ \mathbf{v} &= \frac{d\mathbf{r}}{dt} = \langle 2, -2t \rangle = 2\langle 1, -t \rangle; \\ \frac{ds}{dt} &= \|\mathbf{v}\| = 2(t^2 + 1)^{1/2}; \\ \mathbf{T} &= \frac{d\mathbf{r}}{ds} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = (t^2 + 1)^{-1/2} \langle 1, -t \rangle = \left\langle \frac{1}{\sqrt{t^2 + 1}}, -\frac{t}{\sqrt{t^2 + 1}} \right\rangle; \\ \frac{d\mathbf{T}}{dt} &= -t(t^2 + 1)^{-3/2} \langle 1, -t \rangle + (t^2 + 1)^{-1/2} \langle 0, -1 \rangle = -(t^2 + 1)^{-3/2} \langle t, 1 \rangle; \\ \frac{d\mathbf{T}}{ds} &= \frac{d\mathbf{T}/dt}{ds/dt} = -\frac{1}{2}(t^2 + 1)^{-2} \langle t, 1 \rangle; \\ \left\| \frac{d\mathbf{T}}{ds} \right\| &= (t^2 + 1)^{-1}; \\ \kappa &= \left\| \frac{d\mathbf{T}}{ds} \right\| = \frac{\|d\mathbf{T}/dt\|}{ds/dt} = \frac{1}{2}(t^2 + 1)^{-3/2}; \\ \mathbf{N} &= \frac{d\mathbf{T}/dt}{\|d\mathbf{T}/dt\|} = \frac{d\mathbf{T}/ds}{\|d\mathbf{T}/ds\|} = -(t^2 + 1)^{-1/2} \langle t, 1 \rangle = \left\langle -\frac{t}{\sqrt{t^2 + 1}}, -\frac{1}{\sqrt{t^2 + 1}} \right\rangle. \end{aligned}$$

12.4.7.b

$$\begin{aligned} \mathbf{r} &= \langle t, e^{2t} \rangle; \\ \mathbf{v} &= \frac{d\mathbf{r}}{dt} = \langle 1, 2e^{2t} \rangle; \\ \mathbf{n} &= \mathbf{v} \times \mathbf{v} = \langle -2e^{2t}, 1 \rangle; \\ \|\mathbf{n}\| &= \sqrt{4e^{4t} + 1}; \\ \mathbf{N} &= \frac{\mathbf{n}}{\|\mathbf{n}\|} = \left\langle -\frac{2e^{2t}}{\sqrt{4e^{4t} + 1}}, \frac{1}{\sqrt{4e^{4t} + 1}} \right\rangle. \end{aligned}$$

12.4.9

$$\begin{aligned} \mathbf{r} &= \langle 3 \sin t, 3 \cos t, 4t \rangle; \\ \mathbf{v} &= \frac{d\mathbf{r}}{dt} = \langle 3 \cos t, -3 \sin t, 4 \rangle; \\ \frac{ds}{dt} &= \|\mathbf{v}\| = \sqrt{9 \cos^2 t + 9 \sin^2 t + 16} = 5; \\ \mathbf{T} &= \frac{d\mathbf{r}}{ds} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \left\langle \frac{3}{5} \cos t, -\frac{3}{5} \sin t, \frac{4}{5} \right\rangle; \\ \frac{d\mathbf{T}}{dt} &= \left\langle -\frac{3}{5} \sin t, -\frac{3}{5} \cos t, 0 \right\rangle = -\frac{3}{5} \langle \sin t, \cos t, 0 \rangle; \\ \frac{d\mathbf{T}}{ds} &= \frac{d\mathbf{T}/dt}{ds/dt} = -\frac{3}{25} \langle \sin t, \cos t, 0 \rangle; \\ \left\| \frac{d\mathbf{T}}{ds} \right\| &= \frac{3}{25}; \\ \kappa &= \left\| \frac{d\mathbf{T}}{ds} \right\| = \frac{\|d\mathbf{T}/dt\|}{ds/dt} = \frac{3}{25}; \\ \mathbf{N} &= \frac{d\mathbf{T}/dt}{\|d\mathbf{T}/dt\|} = \frac{d\mathbf{T}/ds}{\|d\mathbf{T}/ds\|} = -\langle \sin t, \cos t, 0 \rangle = \langle -\sin t, -\cos t, 0 \rangle. \end{aligned}$$