

12.6.1 Given $r = a(1 - \cos \theta)$ and $d\theta/dt = 3$:

$$\begin{aligned}\dot{\theta} &= 3; \\ \ddot{\theta} &= 0; \\ \dot{r} &= a \sin \theta \dot{\theta} = 3a \sin \theta; \\ \ddot{r} &= 3a \cos \theta \dot{\theta} = 9a \cos \theta; \\ v_r &= \dot{r} = 3a \sin \theta; \\ v_\theta &= r\dot{\theta} = 3a(1 - \cos \theta); \\ a_r &= \ddot{r} - r\dot{\theta}^2 = 9a(2 \cos \theta - 1); \\ a_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} = 18a \sin \theta.\end{aligned}$$

In other words,

$$\mathbf{v} = 3a \sin \theta \mathbf{u}_r + 3a(1 - \cos \theta) \mathbf{u}_\theta,$$

and

$$\mathbf{a} = 9a(2 \cos \theta - 1) \mathbf{u}_r + 18a \sin \theta \mathbf{u}_\theta.$$

12.6.3 Given $r = e^{a\theta}$ and $d\theta/dt = 2$:

$$\begin{aligned}\dot{\theta} &= 2; \\ \ddot{\theta} &= 0; \\ \dot{r} &= ae^{a\theta} \dot{\theta} = 2ae^{a\theta}; \\ \ddot{r} &= 2a^2 e^{a\theta} \dot{\theta} = 4a^2 e^{a\theta}; \\ v_r &= \dot{r} = 2ae^{a\theta}; \\ v_\theta &= r\dot{\theta} = 2e^{a\theta}; \\ a_r &= \ddot{r} - r\dot{\theta}^2 = 4e^{a\theta}(a^2 - 1); \\ a_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} = 8ae^{a\theta}.\end{aligned}$$

In other words,

$$\mathbf{v} = 2ae^{a\theta} \mathbf{u}_r + 2e^{a\theta} \mathbf{u}_\theta,$$

and

$$\mathbf{a} = 4e^{a\theta}(a^2 - 1) \mathbf{u}_r + 8ae^{a\theta} \mathbf{u}_\theta.$$

12.6.5 Given $r = 2 \cos 4t$ and $\theta = 2t$:

$$\begin{aligned}\dot{\theta} &= 2; \\ \ddot{\theta} &= 0; \\ \dot{r} &= -8 \sin 4t; \\ \ddot{r} &= -32 \cos 4t; \\ v_r &= \dot{r} = -8 \sin 4t; \\ v_\theta &= r\dot{\theta} = 4 \cos 4t; \\ a_r &= \ddot{r} - r\dot{\theta}^2 = -40 \cos 4t; \\ a_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} = -32 \sin 4t.\end{aligned}$$

In other words,

$$\mathbf{v} = -8 \sin 4t \mathbf{u}_r + 4 \cos 4t \mathbf{u}_\theta,$$

and

$$\mathbf{a} = -40 \cos 4t \mathbf{u}_r - 32 \sin 4t \mathbf{u}_\theta.$$