13.6.6 First,

$$\mathrm{d}f(x,y,z) = \mathrm{d}(x^2 - xy - y^2 - z) = 2x\,\mathrm{d}x - y\,\mathrm{d}x - x\,\mathrm{d}y - 2y\,\mathrm{d}y - \mathrm{d}z = (2x - y)\,\mathrm{d}x - (x + 2y)\,\mathrm{d}y - \mathrm{d}z.$$

Passing through $(x, y, z) = (1, 1, -1)$, this is

$$\mathrm{d}x - 3\,\mathrm{d}y - \mathrm{d}z.$$

For the tangent plane, simply change differentials to differences:

$$\begin{gathered}\Delta x - 3\,\Delta y - \Delta z = 0;\\ (x-1) - 3(y-1) - (z+1) = 0;\\ x - 3y - z + 1 = 0;\\ x + 1 = 3y + z.\end{gathered}$$

For the normal line, turn the differential into a gradient and move in that direction:

$$\langle x, y, z\rangle = (1, 1, -1) + t\nabla f(1, 1, -1) = (1, 1, -1) + t\langle 1, -3, -1\rangle = (1 + t, 1 - 3t, -1 - t).$$

In other words,

$$\begin{aligned}&x = t + 1,\\ &y = 1 - 3t, \text{ and}\\ &z = -1 - t.\end{aligned}$$

13.6.29 First,

$$\mathrm{d}f(x,y) = \mathrm{d}(\mathrm{e}^x \cos y) = \cos y\,\mathrm{d}(\mathrm{e}^x) + \mathrm{e}^x\,\mathrm{d}(\cos y) = \mathrm{e}^x \cos y\,\mathrm{d}x - \mathrm{e}^x \sin y\,\mathrm{d}y.$$

By changing differentials to differences, I find the difference of any linearisation:

$$\Delta L = \mathrm{e}^x \cos y\,\Delta x - \mathrm{e}^x \sin y\,\Delta y.$$

a Passing through $(x, y) = (0, 0)$,

$$\Delta L = \mathrm{e}^0 \cos 0(x - 0) - \mathrm{e}^0 \sin 0(y - 0) = x.$$

Since $f(0, 0) = \mathrm{e}^0 \cos 0 = 1$,

$$L = 1 + x = x + 1.$$

b Passing through $(x, y) = (0, \pi/2)$,

$$\Delta L = \mathrm{e}^0 \cos\frac{\pi}{2}(x - 0) - \mathrm{e}^0 \sin\frac{\pi}{2}\left(y - \frac{\pi}{2}\right) = \frac{\pi}{2} - y.$$

Since $f(0, \pi/2) = \mathrm{e}^0 \cos(\pi/2) = 0$,

$$L = 0 + \left(y - \frac{\pi}{2}\right) = y - \frac{\pi}{2}.$$