1 Given

$$\mathbf{r} = (\mathrm{e}^{-t}, 2\cos 3t, 2\sin 3t),$$

find the velocity $\mathbf{v} = \mathrm{d}\mathbf{r}/\mathrm{d}t$ and the acceleration $\mathbf{a} = \mathrm{d}\mathbf{v}/\mathrm{d}t$ when $t = 0$.

2 Given

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \langle t^3 + 4t, t, 2t^2 \rangle$$

and $\mathbf{r} = (1, 1, 0)$ when $t = 0$, find $\mathbf{r}$ as a function of t.

3 Given

$$\mathbf{r} = (12\sin t, -12\cos t, 5t),$$

find the length of the curve from $t = 0$ to $t = 2\pi$.

4 Find

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2 + y^4}.$$

5 Find the (first) partial derivatives of

$$f(x, y) = x^2\mathrm{e}^{-y}$$

when $(x, y) = (0, 1)$.

6 Given

$$f(x, y, z) = \ln(xy + xz + xy),$$

find the gradient $\nabla f(1, 1, 1)$.

7 Given

$$u = x^2 + y^2,$$

find the largest value of u subject to the condition that

$$4x^2 + 9y^2 = 36.$$

8 Write the integral of $f(x, y)$ on the triangle bounded by $x = 0$, $y = 0$, and $2x + 3y = 4$ as an iterated integral.

9 Write

$$\int_0^1 \int_0^x \int_0^{x+y} (x + y + z)\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$$

as a sum of iterated integrals ending in $\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$.

10 Write the integral of $3r^2\cos\theta$, on the interior of the circle with radius 4 centred at the origin, as an iterated integral in polar coordinates.

11 Write

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (x^2 + y^2 + z^2)\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$$

as an iterated integral in spherical coordinates.

12 If C is the oriented curve given by

$$\mathbf{r} = (t, t^2, t^3)$$

for $0 \leq t \leq 1$, write

$$\int_C (x\,\mathrm{d}y + y\,\mathrm{d}z + z\,\mathrm{d}x)$$

as an ordinary integral in t.

13 Write the flux of

$$\mathbf{F} = \langle x, y \rangle$$

across the curve given by

$$x^2 + y^2 = 1$$

for $x, y \geq 0$, in the direction away from the origin, as an ordinary integral in the polar coordinate θ.

14 Write an integral for the surface area of the ellipsoid

$$x^2 + 4y^2 + 9z^2 = 36$$

parametrised by modified spherical coordinates.

15 **Extra credit:** Find the curl and divergence of

$$\mathbf{F}(x, y, z) = \left\langle 2xyz, \frac{x}{y^2}, \sin x \cos y \right\rangle.$$

Answers

1 $\mathbf{v} = \langle -1, 0, 6 \rangle,\ \mathbf{a} = \langle 1, -18, 0 \rangle$

2 $\mathbf{r} = \left(\frac{1}{4}t^4 + 2t^2 + 1, \frac{1}{2}t^2 + 1, \frac{2}{3}t^3 \right)$

3 26π

4 undefined (does not exist)

5 $f_1(0,1) = 0,\ f_2(0,1) = 0$

6 $\left\langle 1, \frac{2}{3}, \frac{1}{3} \right\rangle$

7 9

8 $\int_0^2 \int_0^{4/3-2x/3} f(x,y)\,\mathrm{d}y\,\mathrm{d}x$ or $\int_0^{4/3} \int_0^{2-3y/2} f(x,y)\,\mathrm{d}x\,\mathrm{d}y$

9 $\int_0^1 \int_0^{z/2} \int_{z-y}^1 (x+y+z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z + \int_1^2 \int_{z-1}^{z/2} \int_{z-y}^1 (x+y+z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z + \int_0^2 \int_{z/2}^1 \int_y^1 (x+y+z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

10 $\int_0^{2\pi} \int_0^4 3r^3 \cos\theta\,\mathrm{d}r\,\mathrm{d}\theta$

11 $\int_0^{2\pi} \int_0^{\pi} \int_0^1 \rho^3 \sin\phi\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$

12 $\int_0^1 (3t^4 + t^3 + 2t^2)\,\mathrm{d}t$

13 $\int_0^{\pi/2} \mathrm{d}\theta$

14 $\int_0^{2\pi}\int_0^{\pi} 6\sqrt{9 + 3\sin^2\phi\sin^2\theta - 8\sin^2\phi}\,\sin\phi\,\mathrm{d}\phi\,\mathrm{d}\theta$

15 $\nabla \times \mathbf{F}(x,y,z) = \left\langle -\sin x \sin y, 2xy - \cos x \cos y, \frac{1}{y^2} - 2xz \right\rangle,\ \nabla \cdot \mathbf{F}(x,y,z) = 2yz - \frac{2x}{y^3}$