

3.3.10

$$\begin{aligned}
 df(x, y) &= d\left(\frac{x}{x^2 + y^2}\right) = \frac{(x^2 + y^2) dx - x d(x^2 + y^2)}{(x^2 + y^2)^2} = \frac{x^2 dx + y^2 dx - x(d(x^2) + d(y^2))}{(x^2 + y^2)^2} \\
 &= \frac{x^2 dx + y^2 dx - x(2x^{2-1} dx) - x(2y^{2-1} dy)}{(x^2 + y^2)^2} = \frac{x^2 dx + y^2 dx - 2x^2 dx - 2xy dy}{(x^2 + y^2)^2} \\
 &= \frac{y^2 - x^2}{(x^2 + y^2)^2} dx - \frac{2xy}{(x^2 + y^2)^2} dy.
 \end{aligned}$$

Therefore,

$$f_1(x, y) = \frac{\partial f(x, y)}{\partial x} = \frac{y^2 - x^2}{(x^2 + y^2)^2},$$

and

$$f_2(x, y) = \frac{\partial f(x, y)}{\partial y} = -\frac{2xy}{(x^2 + y^2)^2}.$$

3.3.43

$$\begin{aligned}
 g_1(x, y) &= (\partial/\partial x)g(x, y) = 2xy + y \cos x, \\
 g_{1,1}(x, y) &= (\partial/\partial x)^2 g(x, y) = 2y - y \sin x, \\
 g_{1,2}(x, y) &= (\partial/\partial y)(\partial/\partial x)g(x, y) = 2x + \cos x; \\
 g_2(x, y) &= \frac{\partial g(x, y)}{\partial y} = x^2 - \sin y + \sin x; \\
 g_{2,1}(x, y) &= (\partial/\partial x)(\partial/\partial y)g(x, y) = 2x + \cos x, \\
 g_{2,2}(x, y) &= (\partial/\partial y)^2 g(x, y) = -\cos y.
 \end{aligned}$$