1 Given

$$\mathbf{r} = (t^2, 3t, 5),$$

find the velocity $\mathbf{v} = \mathrm{d}\mathbf{r}/\mathrm{d}t$ and the acceleration $\mathbf{a} = \mathrm{d}\mathbf{v}/\mathrm{d}t$ when $t = 2$.

a $\mathbf{v} = \langle 4, 6, 5\rangle,\ \mathbf{a} = \langle 2, 0, 5\rangle$

b $\mathbf{v} = \langle 4, 3, 0\rangle,\ \mathbf{a} = \langle 2, 0, 0\rangle$

c $\mathbf{v} = \langle 4, 3, 0\rangle,\ \mathbf{a} = \langle 2, 0, 5\rangle$

d $\mathbf{v} = \langle 4, 6, 5\rangle,\ \mathbf{a} = \langle 4, 3, 0\rangle$

2 Given

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \langle t^2, 3t, 5\rangle$$

and $\mathbf{r} = (0, 0, 1)$ when $t = 0$, find $\mathbf{r}$ as a function of t.

a $\mathbf{r} = (0, 0, -3t^2 + 1)$

b $\mathbf{r} = (0, 0, 3t^2 + 1)$

c $\mathbf{r} = \left(\frac{1}{3}t^3, \frac{3}{2}t^2, 5t\right)$

d $\mathbf{r} = \left(\frac{1}{3}t^3, \frac{3}{2}t^2, 5t + 1\right)$

3 Find

$$\lim_{(x,y)\to(0,0)} \frac{x^2 + y^2}{x^2y^2 + y^4}.$$

a undefined (does not exist)

b 2

c 1

d 0

4 Find the (first) partial derivatives of

$$f(x, y) = x^2 + 2y^2$$

when $(x, y) = (1, 1/2)$.

a $f_1(x, y) = \frac{\partial f(x, y)}{\partial x} = 5,\ f_2(x, y) = \frac{\partial f(x, y)}{\partial y} = 5$

b $f_1(x, y) = \frac{\partial f(x, y)}{\partial x} = 1,\ f_2(x, y) = \frac{\partial f(x, y)}{\partial y} = \frac{1}{2}$

c $f_1(x, y) = \frac{\partial f(x, y)}{\partial x} = 2,\ f_2(x, y) = \frac{\partial f(x, y)}{\partial y} = 2$

d $f_1(x, y) = \frac{\partial f(x, y)}{\partial x} = \frac{5}{2},\ f_2(x, y) = \frac{\partial f(x, y)}{\partial y} = 3$

5 Given

$$f(x,y) = y^2 \cos(3x),$$

find the gradient $\nabla f(\pi/4, 1)$.

a $\left\langle \frac{3\sqrt{2}}{2}, \sqrt{2} \right\rangle$

b $\left\langle -\frac{3\sqrt{2}}{2}, -\sqrt{2} \right\rangle$

c $\left\langle -\frac{3\sqrt{2}}{2}, \sqrt{2} \right\rangle$

d $\left\langle \frac{3\sqrt{2}}{2}, -\sqrt{2} \right\rangle$

6 Given

$$u = 2x^4 + 2y^4 - 9x^2 + 3y^2,$$

find the minimum value of u.

a $-\frac{729}{8}$

b none (or $-\infty$)

c $-\frac{81}{8}$

d 0

7 If C is the oriented curve given by

$$\mathbf{r} = (t, t^2, t^3)$$

for $0 \le t \le 1$, write

$$\int_C (y\,\mathrm{d}x + x\,\mathrm{d}y + z\,\mathrm{d}y - y\,\mathrm{d}z)$$

as an ordinary integral in t.

a $\int_0^1 (t^4 + 3t^2)\,\mathrm{d}t$

b $\int_0^1 (-t^4 + 3t^2)\,\mathrm{d}t$

c $\int_0^1 (4t)\,\mathrm{d}t$

d $\int_0^1 (2t^3)\,\mathrm{d}t$

8 Write the flux of

$$\mathbf{F} = \langle x + 2y, -x \rangle$$

across the curve given by

$$x^2 + y^2 = 1,$$

in the direction away from the origin, as an ordinary integral in the polar coordinate θ.

a $\int_0^{2\pi} (\cos^2\theta + \sin\theta\cos\theta)\,\mathrm{d}\theta$

b $\int_0^{2\pi} (-\sin^2\theta - \sin\theta\cos\theta - 1)\,\mathrm{d}\theta$

c $\int_0^{2\pi} (\sin^2\theta + \sin\theta\cos\theta + 1)\,\mathrm{d}\theta$

d $\int_0^{2\pi} (-\cos^2\theta - \sin\theta\cos\theta)\,\mathrm{d}\theta$

9 Given

$$\mathbf{r} = (2t, 3t, 6t),$$

find the length of the curve from $t = 0$ to $t = 2\pi$.

a 7π

b 14

c 7

d 14π

10 Write

$$\int_0^2 \int_y^2 \mathrm{e}^{-x^2}\, \mathrm{d}x\, \mathrm{d}y$$

as an iterated integral ending in $\mathrm{d}y\, \mathrm{d}x$.

a $\int_0^2 \int_0^x \mathrm{e}^{-x^2}\, \mathrm{d}y\, \mathrm{d}x$

b $\int_y^2 \int_0^2 \mathrm{e}^{-x^2}\, \mathrm{d}y\, \mathrm{d}x$

c $\int_0^2 \int_x^2 \mathrm{e}^{-x^2}\, \mathrm{d}y\, \mathrm{d}x$

d $\int_0^y \int_0^2 \mathrm{e}^{-x^2}\, \mathrm{d}y\, \mathrm{d}x$

11 Write the volume of the pyramid bounded by $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$ as an iterated integral ending in $\mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z$.

a $\int_0^1 \int_0^1 \int_0^{1-y-z} \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z$

b $\int_0^1 \int_0^{1-z} \int_0^{1-z} \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z$

c $\int_0^1 \int_0^1 \int_0^{1-z} \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z$

d $\int_0^1 \int_0^{1-z} \int_0^{1-y-z} \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z$

12 Write the integral of $3r^2 \cos\theta$, on the region above the horizontal axis with a distance between 2 and 5 from the origin, as an iterated integral in polar coordinates.

a $\int_0^{2\pi} \int_2^5 3r^3 \cos\theta\, \mathrm{d}r\, \mathrm{d}\theta$

b $\int_0^{2\pi} \int_2^5 3r \cos\theta\, \mathrm{d}r\, \mathrm{d}\theta$

c $\int_0^{\pi} \int_2^5 3r^3 \cos\theta\, \mathrm{d}r\, \mathrm{d}\theta$

d $\int_0^{\pi} \int_2^5 3r \cos\theta\, \mathrm{d}r\, \mathrm{d}\theta$

13 Write

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{9-x^2-y^2} (x^2+y^2+z^2)\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$$

as an iterated integral in cylindrical coordinates.

a $\int_0^{\pi/2} \int_0^2 \int_0^{9-4r^2} (r^2+z^2)\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

b $\int_0^{\pi/2} \int_0^2 \int_0^{9-r^2} (r^3+rz^2)\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

c $\int_0^{\pi/2} \int_0^2 \int_0^{9-4r^2} (r^3+rz^2)\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

d $\int_0^{\pi/2} \int_0^2 \int_0^{9-r^2} (r^2+z^2)\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

14 Write an iterated integral for the surface area of the paraboloid

$$z = x^2 + y^2,$$

for $z \leq 4$, parametrised by the cylindrical coordinates r and θ.

a $\int_0^{2\pi} \int_0^4 r\sqrt{4r^2+1}\,\mathrm{d}r\,\mathrm{d}\theta$

b $\int_0^{2\pi} \int_0^2 \sqrt{4r^2+1}\,\mathrm{d}r\,\mathrm{d}\theta$

c $\int_0^{2\pi} \int_0^2 r\sqrt{4r^2+1}\,\mathrm{d}r\,\mathrm{d}\theta$

d $\int_0^{2\pi} \int_0^4 \sqrt{4r^2+1}\,\mathrm{d}r\,\mathrm{d}\theta$

15 Find the curl of

$$\mathbf{F}(x,y,z) = \langle 2xyz, -3xz^3, yz^2 \rangle.$$

a $\nabla \times \mathbf{F}(x,y,z) = \langle 9xz^2 - z^2, -2xy, 3z^3 - 2xz \rangle$

b $\nabla \times \mathbf{F}(x,y,z) = \langle -9xz^2 + z^2, 2xy, -3z^3 + 2xz \rangle$

c $\nabla \times \mathbf{F}(x,y,z) = \langle 9xz^2 + z^2, 2xy, -3z^3 - 2xz \rangle$

d $\nabla \times \mathbf{F}(x,y,z) = \langle -9xz^2 - z^2, -2xy, 3z^3 + 2xz \rangle$

16 Find the divergence of

$$\mathbf{F}(x,y,z) = \langle 2xyz, -3xz^3, yz^2 \rangle.$$

a $\nabla \cdot \mathbf{F}(x,y,z) = 4yz$

b $\nabla \cdot \mathbf{F}(x,y,z) = 0$

c $\nabla \cdot \mathbf{F}(x,y,z) = \langle 2yz, 0, 2yz \rangle$

d $\nabla \cdot \mathbf{F}(x,y,z) = \langle 2yz, 0, -2yz \rangle$

Answers

1 B, 2 D, 3 A, 4 C, 5 B, 6 C, 7 B, 8 A, 9 D, 10 A, 11 D, 12 C, 13 B, 14 C, 15 C, 16 A