1 Given
$$\mathbf{r} = \langle t^2, 3t, 5\rangle = t^2\mathbf{i} + 3t\mathbf{j} + 5\mathbf{k},$$
find the velocity $\mathbf{v} = \mathrm{d}\mathbf{r}/\mathrm{d}t$ and the acceleration $\mathbf{a} = \mathrm{d}\mathbf{v}/\mathrm{d}t$ when $t = 2$.

a $\mathbf{v} = \langle 4, 6, 5\rangle = 4\mathbf{i} + 6\mathbf{j} + 5\mathbf{k},\ \mathbf{a} = \langle 2, 0, 5\rangle = 2\mathbf{i} + 5\mathbf{k}$

b $\mathbf{v} = \langle 4, 3, 0\rangle = 4\mathbf{i} + 3\mathbf{j},\ \mathbf{a} = \langle 2, 0, 0\rangle = 2\mathbf{i}$

c $\mathbf{v} = \langle 4, 3, 0\rangle = 4\mathbf{i} + 3\mathbf{j},\ \mathbf{a} = \langle 2, 0, 5\rangle = 2\mathbf{i} + 5\mathbf{k}$

d $\mathbf{v} = \langle 4, 6, 5\rangle = 4\mathbf{i} + 6\mathbf{j} + 5\mathbf{k},\ \mathbf{a} = \langle 4, 3, 0\rangle = 4\mathbf{i} + 3\mathbf{j}$

2 Given
$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \langle t^2, 3t, 5\rangle = t^2\mathbf{i} + 3t\mathbf{j} + 5\mathbf{k}$$
and $\mathbf{r} = \langle 0, 0, 1\rangle = \mathbf{k}$ when $t = 0$, find $\mathbf{r}$ as a function of t.

a $\mathbf{r} = \langle 0, 0, -3t^2 + 1\rangle = (-3t^2 + 1)\mathbf{k}$

b $\mathbf{r} = \langle 0, 0, 3t^2 + 1\rangle = (3t^2 + 1)\mathbf{k}$

c $\mathbf{r} = \left\langle \frac{1}{3}t^3, \frac{3}{2}t^2, 5t\right\rangle = \frac{1}{3}t^3\mathbf{i} + \frac{3}{2}t^2\mathbf{j} + 5t\mathbf{k}$

d $\mathbf{r} = \left\langle \frac{1}{3}t^3, \frac{3}{2}t^2, 5t + 1\right\rangle = \frac{1}{3}t^3\mathbf{i} + \frac{3}{2}t^2\mathbf{j} + (5t + 1)\mathbf{k}$

3 Find
$$\lim_{(x,y)\to(0,0)} \frac{x^2 + y^2}{x^2y^2 + y^4}.$$

a undefined (does not exist)

b 2

c 1

d 0

4 Find the (first) partial derivatives of
$$f(x, y) = x^2 + 2y^2$$
when $(x, y) = (1, 1/2)$.

a $D_1 f(x, y) = \frac{\partial f(x, y)}{\partial x} = 5,\ D_2 f(x, y) = \frac{\partial f(x, y)}{\partial y} = 5$

b $D_1 f(x, y) = \frac{\partial f(x, y)}{\partial x} = 1,\ D_2 f(x, y) = \frac{\partial f(x, y)}{\partial y} = \frac{1}{2}$

c $D_1 f(x, y) = \frac{\partial f(x, y)}{\partial x} = 2,\ D_2 f(x, y) = \frac{\partial f(x, y)}{\partial y} = 2$

d $D_1 f(x, y) = \frac{\partial f(x, y)}{\partial x} = \frac{5}{2},\ D_2 f(x, y) = \frac{\partial f(x, y)}{\partial y} = 3$

5 Given

$$f(x, y) = y^2 \cos(3x),$$

find the gradient $\nabla f(\pi/4, 1)$.

a $\left\langle \frac{3\sqrt{2}}{2}, \sqrt{2} \right\rangle = \frac{3\sqrt{2}}{2}\,\mathbf{i} + \sqrt{2}\,\mathbf{j}$

b $\left\langle -\frac{3\sqrt{2}}{2}, -\sqrt{2} \right\rangle = -\frac{3\sqrt{2}}{2}\,\mathbf{i} - \sqrt{2}\,\mathbf{j}$

c $\left\langle -\frac{3\sqrt{2}}{2}, \sqrt{2} \right\rangle = -\frac{3\sqrt{2}}{2}\,\mathbf{i} + \sqrt{2}\,\mathbf{j}$

d $\left\langle \frac{3\sqrt{2}}{2}, -\sqrt{2} \right\rangle = \frac{3\sqrt{2}}{2}\,\mathbf{i} - \sqrt{2}\,\mathbf{j}$

6 Given

$$u = 2x^4 + 2y^4 - 9x^2 + 3y^2,$$

find the minimum value of u.

a $-\frac{729}{8}$

b none (or $-\infty$)

c $-\frac{81}{8}$

d 0

7 If C is the oriented curve given by

$$x = t,\ y = t^2,\ z = t^3,\ 0 \le t \le 1,$$

write

$$\int_C (y\,\mathrm{d}x + x\,\mathrm{d}y + z\,\mathrm{d}y - y\,\mathrm{d}z)$$

as an ordinary integral in t.

a $\int_0^1 (t^4 + 3t^2)\,\mathrm{d}t$

b $\int_0^1 (-t^4 + 3t^2)\,\mathrm{d}t$

c $\int_0^1 (4t)\,\mathrm{d}t$

d $\int_0^1 (2t^3)\,\mathrm{d}t$

8 Write the flux of

$$\mathbf{F} = \langle x + 2y, -x \rangle = (x + 2y)\mathbf{i} - x\mathbf{j}$$

across the curve given by

$$x^2 + y^2 = 1,$$

in the direction away from the origin, as an ordinary integral in the polar coordinate θ.

a $\int_0^{2\pi} (\cos^2\theta + \sin\theta\cos\theta)\,\mathrm{d}\theta$

b $\int_0^{2\pi} (-\sin^2\theta - \sin\theta\cos\theta - 1)\,\mathrm{d}\theta$

c $\int_0^{2\pi} (\sin^2\theta + \sin\theta\cos\theta + 1)\,\mathrm{d}\theta$

d $\int_0^{2\pi} (-\cos^2\theta - \sin\theta\cos\theta)\,\mathrm{d}\theta$

9 Given

$$\mathbf{r} = \langle 2t, 3t, 6t \rangle = 2t\mathbf{i} + 3t\mathbf{j} + 6t\mathbf{k},$$

find the length of the curve from $t = 0$ to $t = 2\pi$.

a 7π

b 14

c 7

d 14π

10 Write

$$\int_0^2 \int_y^2 \mathrm{e}^{-x^2}\,\mathrm{d}x\,\mathrm{d}y$$

as an iterated integral ending in $\mathrm{d}y\,\mathrm{d}x$.

a $\int_0^2 \int_0^x \mathrm{e}^{-x^2}\,\mathrm{d}y\,\mathrm{d}x$

b $\int_y^2 \int_0^2 \mathrm{e}^{-x^2}\,\mathrm{d}y\,\mathrm{d}x$

c $\int_0^2 \int_x^2 \mathrm{e}^{-x^2}\,\mathrm{d}y\,\mathrm{d}x$

d $\int_0^y \int_0^2 \mathrm{e}^{-x^2}\,\mathrm{d}y\,\mathrm{d}x$

11 Write the volume of the pyramid bounded by $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$ as an iterated integral ending in $\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$.

a $\int_0^1 \int_0^1 \int_0^{1-y-z} \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

b $\int_0^1 \int_0^{1-z} \int_0^{1-z} \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

c $\int_0^1 \int_0^1 \int_0^{1-z} \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

d $\int_0^1 \int_0^{1-z} \int_0^{1-y-z} \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

12 Write the integral of $3r^2 \cos\theta$, on the region above the horizontal axis with a distance between 2 and 5 from the origin, as an iterated integral in polar coordinates.

a $\int_0^{2\pi} \int_2^5 3r^3 \cos\theta\,\mathrm{d}r\,\mathrm{d}\theta$

b $\int_0^{2\pi} \int_2^5 3r \cos\theta\,\mathrm{d}r\,\mathrm{d}\theta$

c $\int_0^{\pi} \int_2^5 3r^3 \cos\theta\,\mathrm{d}r\,\mathrm{d}\theta$

d $\int_0^{\pi} \int_2^5 3r \cos\theta\,\mathrm{d}r\,\mathrm{d}\theta$

13 Write

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{9-x^2-y^2} (x^2+y^2+z^2)\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$$

as an iterated integral in cylindrical coordinates.

a $\int_0^{\pi/2} \int_0^2 \int_0^{9-4r^2} (r^2+z^2)\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

b $\int_0^{\pi/2} \int_0^2 \int_0^{9-r^2} (r^3+rz^2)\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

c $\int_0^{\pi/2} \int_0^2 \int_0^{9-4r^2} (r^3+rz^2)\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

d $\int_0^{\pi/2} \int_0^2 \int_0^{9-r^2} (r^2+z^2)\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

14 Write an iterated integral for the surface area of the paraboloid

$$z = x^2 + y^2,$$

for $z \le 4$, parametrised by the cylindrical coordinates r and θ.

a $\int_0^{2\pi} \int_0^4 r\sqrt{4r^2+1}\,\mathrm{d}r\,\mathrm{d}\theta$

b $\int_0^{2\pi} \int_0^2 \sqrt{4r^2+1}\,\mathrm{d}r\,\mathrm{d}\theta$

c $\int_0^{2\pi} \int_0^2 r\sqrt{4r^2+1}\,\mathrm{d}r\,\mathrm{d}\theta$

d $\int_0^{2\pi} \int_0^4 \sqrt{4r^2+1}\,\mathrm{d}r\,\mathrm{d}\theta$

15 Find the curl of

$$\mathbf{F}(x,y,z) = \langle 2xyz, -3xz^3, yz^2\rangle = 2xyz\mathbf{i} - 3xz^3\mathbf{j} + yz^2\mathbf{k}.$$

a $\nabla \times \mathbf{F}(x,y,z) = \langle 9xz^2 - z^2, -2xy, 3z^3 - 2xz\rangle = (9xz^2 - z^2)\mathbf{i} - 2xy\mathbf{j} + (3z^3 - 2xz)\mathbf{k}$

b $\nabla \times \mathbf{F}(x,y,z) = \langle -9xz^2 + z^2, 2xy, -3z^3 + 2xz\rangle = (-9xz^2 + z^2)\mathbf{i} + 2xy\mathbf{j} + (-3z^3 + 2xz)\mathbf{k}$

c $\nabla \times \mathbf{F}(x,y,z) = \langle 9xz^2 + z^2, 2xy, -3z^3 - 2xz\rangle = (9xz^2 + z^2)\mathbf{i} + 2xy\mathbf{j} + (-3z^3 - 2xz)\mathbf{k}$

d $\nabla \times \mathbf{F}(x,y,z) = \langle -9xz^2 - z^2, -2xy, 3z^3 + 2xz\rangle = (-9xz^2 - z^2)\mathbf{i} - 2xy\mathbf{j} + (3z^3 + 2xz)\mathbf{k}$

16 If C is the oriented curve given by

$$x = \cos t,\ y = \sin t,\ 0 \le t \le 2\pi,$$

use Green's Theorem to write

$$\int_C (x\,\mathrm{d}x + 2y\,\mathrm{d}x - x\,\mathrm{d}y)$$

as a double integral (with respect to area) over the region R where $x^2 + y^2 \le 1$.

a $\iint (-3)\,\mathrm{d}A$

b $\iint 3\,\mathrm{d}A$

c $\iint 3xy\,\mathrm{d}A$

d $\iint (-3)xy\,\mathrm{d}A$

Answers

1 B, 2 D, 3 A, 4 C, 5 B, 6 C, 7 B, 8 A, 9 D, 10 A, 11 D, 12 C, 13 B, 14 C, 15 C, 16 A