

15.1.22 From the given parametrization of the curve C , we have

$$\begin{aligned}x &= \cos t, \\y &= \sin t, \\0 &\leq t \leq 2\pi.\end{aligned}$$

Differentiating,

$$\begin{aligned}dx &= -\sin t \, dt, \\dy &= \cos t \, dt,\end{aligned}$$

so

$$ds = \|d\mathbf{r}\| = \|d(x, y)\| = \|\langle dx, dy \rangle\| = \sqrt{dx^2 + dy^2} = \sqrt{(-\sin t \, dt)^2 + (\cos t \, dt)^2} = |dt|.$$

Therefore,

$$\begin{aligned}\int_C (x - y + 3) \, ds &= \int_{t=0}^{2\pi} ((\cos t) - (\sin t) + 3) |dt| = \int_0^{2\pi} (\cos t - \sin t + 3) \, dt = (\sin t + \cos t + 3t) \Big|_{t=0}^{2\pi} \\&= (\sin(2\pi) + \cos(2\pi) + 3(2\pi)) - (\sin(0) + \cos(0) + 3(0)) \\&= (0 + 1 + 6\pi) - (0 + 1 + 0) = (6\pi + 1) - (1) = 6\pi.\end{aligned}$$

You can also guess this answer geometrically, since C is the circle of radius 1 centred at the origin. Both x and y are completely balanced between positive and negative around this circle (since it is centred at the origin), so the integrals of $x \, ds$ and of $y \, ds$ will be zero. As for the integral of $3 \, ds$, this is simply 3 times the circumference of the circle, which is 2π (since it has radius 1). Thus, the final answer is

$$0 - 0 + 3(2\pi) = 6\pi.$$