There will be a comprehensive final exam taken in class on March 19 Thursday. The exam will be multiple choice, with no partial credit (except possibly on extra credit problems).

For the exam, you may use one sheet of notes that you wrote yourself. However, you may not use your book or anything else not written by you. You certainly should not talk to other people! Calculators are allowed, although you shouldn't really need them, but not communication devices (like cell phones).

1 Given

$$\mathbf{r} = \langle t^2, 3t, 5\rangle = t^2\mathbf{i} + 3t\mathbf{j} + 5\mathbf{k},$$

find the velocity $\mathbf{v} = \mathrm{d}\mathbf{r}/\mathrm{d}t$ and the acceleration $\mathbf{a} = \mathrm{d}\mathbf{v}/\mathrm{d}t$ when $t = 2$.

a $\mathbf{v} = \langle 4, 6, 5\rangle = 4\mathbf{i} + 6\mathbf{j} + 5\mathbf{k},\ \mathbf{a} = \langle 2, 0, 5\rangle = 2\mathbf{i} + 5\mathbf{k}$

b $\mathbf{v} = \langle 4, 3, 0\rangle = 4\mathbf{i} + 3\mathbf{j},\ \mathbf{a} = \langle 2, 0, 0\rangle = 2\mathbf{i}$

c $\mathbf{v} = \langle 4, 3, 0\rangle = 4\mathbf{i} + 3\mathbf{j},\ \mathbf{a} = \langle 2, 0, 5\rangle = 2\mathbf{i} + 5\mathbf{k}$

d $\mathbf{v} = \langle 4, 6, 5\rangle = 4\mathbf{i} + 6\mathbf{j} + 5\mathbf{k},\ \mathbf{a} = \langle 4, 3, 0\rangle = 4\mathbf{i} + 3\mathbf{j}$

2 Given

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \langle t^2, 3t, 5\rangle = t^2\mathbf{i} + 3t\mathbf{j} + 5\mathbf{k}$$

and $\mathbf{r} = \langle 0, 0, 1\rangle = \mathbf{k}$ when $t = 0$, find $\mathbf{r}$ as a function of t.

a $\mathbf{r} = \langle 0, 0, -3t^2 + 1\rangle = (-3t^2 + 1)\mathbf{k}$

b $\mathbf{r} = \langle 0, 0, 3t^2 + 1\rangle = (3t^2 + 1)\mathbf{k}$

c $\mathbf{r} = \left\langle \frac{1}{3}t^3, \frac{3}{2}t^2, 5t\right\rangle = \frac{1}{3}t^3\mathbf{i} + \frac{3}{2}t^2\mathbf{j} + 5t\mathbf{k}$

d $\mathbf{r} = \left\langle \frac{1}{3}t^3, \frac{3}{2}t^2, 5t + 1\right\rangle = \frac{1}{3}t^3\mathbf{i} + \frac{3}{2}t^2\mathbf{j} + (5t + 1)\mathbf{k}$

3 Find

$$\lim_{(x,y)\to(0,0)} \frac{x^2 + y^2}{x^2y^2 + y^4}.$$

a undefined (does not exist)

b 2

c 1

d 0

4 If C is the oriented curve given by

$$x = t,\ y = t^2,\ z = t^3,\ 0 \le t \le 1,$$

write

$$\int_C (y\,\mathrm{d}x + x\,\mathrm{d}y + z\,\mathrm{d}y - y\,\mathrm{d}z)$$

as an ordinary integral in t.

a $\int_0^1 (t^4 + 3t^2)\,\mathrm{d}t$

b $\int_0^1 (-t^4 + 3t^2)\,\mathrm{d}t$

c $\int_0^1 (4t)\,\mathrm{d}t$

d $\int_0^1 (2t^3)\,\mathrm{d}t$

5 Write the flux of

$$\mathbf{F} = \langle x+2y, -x\rangle = (x+2y)\mathbf{i} - x\mathbf{j}$$

across the curve given by

$$x^2 + y^2 = 1,$$

in the direction away from the origin, as an ordinary integral in the polar coordinate θ.

a $\int_0^{2\pi} (\cos^2\theta + \sin\theta\cos\theta)\,\mathrm{d}\theta$

b $\int_0^{2\pi} (-\sin^2\theta - \sin\theta\cos\theta - 1)\,\mathrm{d}\theta$

c $\int_0^{2\pi} (\sin^2\theta + \sin\theta\cos\theta + 1)\,\mathrm{d}\theta$

d $\int_0^{2\pi} (-\cos^2\theta - \sin\theta\cos\theta)\,\mathrm{d}\theta$

6 Given

$$\mathbf{r} = \langle 2t, 3t, 6t\rangle = 2t\mathbf{i} + 3t\mathbf{j} + 6t\mathbf{k},$$

find the length of the curve from $t = 0$ to $t = 2\pi$.

a 7π

b 14

c 7

d 14π

7 Find the (first) partial derivatives of

$$f(x,y) = x^2 + 2y^2$$

when $(x,y) = (1, 1/2)$.

a $D_1 f(x,y) = \dfrac{\partial f(x,y)}{\partial x} = 5,\ D_2 f(x,y) = \dfrac{\partial f(x,y)}{\partial y} = 5$

b $D_1 f(x,y) = \dfrac{\partial f(x,y)}{\partial x} = 1,\ D_2 f(x,y) = \dfrac{\partial f(x,y)}{\partial y} = \dfrac{1}{2}$

c $D_1 f(x,y) = \dfrac{\partial f(x,y)}{\partial x} = 2,\ D_2 f(x,y) = \dfrac{\partial f(x,y)}{\partial y} = 2$

d $D_1 f(x,y) = \dfrac{\partial f(x,y)}{\partial x} = \dfrac{5}{2},\ D_2 f(x,y) = \dfrac{\partial f(x,y)}{\partial y} = 3$

8 Given

$$f(x,y) = y^2\cos(3x),$$

find the gradient $\nabla f(\pi/4, 1)$.

a $\left\langle \dfrac{3\sqrt{2}}{2}, \sqrt{2}\right\rangle = \dfrac{3\sqrt{2}}{2}\,\mathbf{i} + \sqrt{2}\,\mathbf{j}$

b $\left\langle -\dfrac{3\sqrt{2}}{2}, -\sqrt{2}\right\rangle = -\dfrac{3\sqrt{2}}{2}\,\mathbf{i} - \sqrt{2}\,\mathbf{j}$

c $\left\langle -\dfrac{3\sqrt{2}}{2}, \sqrt{2}\right\rangle = -\dfrac{3\sqrt{2}}{2}\,\mathbf{i} + \sqrt{2}\,\mathbf{j}$

d $\left\langle \dfrac{3\sqrt{2}}{2}, -\sqrt{2}\right\rangle = \dfrac{3\sqrt{2}}{2}\,\mathbf{i} - \sqrt{2}\,\mathbf{j}$

9 Given

$$u = 2x^4 + 2y^4 - 9x^2 + 3y^2,$$

find the minimum value of u.

a $-\dfrac{729}{8}$

b none (or $-\infty$)

c $-\dfrac{81}{8}$

d 0

10 Write

$$\int_0^2 \int_y^2 e^{-x^2} \, dx \, dy$$

as an iterated integral ending in $dy\,dx$.

a $\int_0^2 \int_0^x e^{-x^2} \, dy \, dx$

b $\int_y^2 \int_0^2 e^{-x^2} \, dy \, dx$

c $\int_0^2 \int_x^2 e^{-x^2} \, dy \, dx$

d $\int_0^y \int_0^2 e^{-x^2} \, dy \, dx$

11 Write the volume of the pyramid bounded by $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$ as an iterated integral ending in $dx\,dy\,dz$.

a $\int_0^1 \int_0^1 \int_0^{1-y-z} dx \, dy \, dz$

b $\int_0^1 \int_0^{1-z} \int_0^{1-z} dx \, dy \, dz$

c $\int_0^1 \int_0^1 \int_0^{1-z} dx \, dy \, dz$

d $\int_0^1 \int_0^{1-z} \int_0^{1-y-z} dx \, dy \, dz$

12 Write the integral of $3r^2 \cos\theta$, on the region above the horizontal axis with a distance between 2 and 5 from the origin, as an iterated integral in polar coordinates.

a $\int_0^{2\pi} \int_2^5 3r^3 \cos\theta \, dr \, d\theta$

b $\int_0^{2\pi} \int_2^5 3r \cos\theta \, dr \, d\theta$

c $\int_0^{\pi} \int_2^5 3r^3 \cos\theta \, dr \, d\theta$

d $\int_0^{\pi} \int_2^5 3r \cos\theta \, dr \, d\theta$

13 Write

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{9-x^2-y^2} (x^2+y^2+z^2)\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$$

as an iterated integral in cylindrical coordinates.

a $\int_0^{\pi/2} \int_0^2 \int_0^{9-4r^2} (r^2+z^2)\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

b $\int_0^{\pi/2} \int_0^2 \int_0^{9-r^2} (r^3+rz^2)\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

c $\int_0^{\pi/2} \int_0^2 \int_0^{9-4r^2} (r^3+rz^2)\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

d $\int_0^{\pi/2} \int_0^2 \int_0^{9-r^2} (r^2+z^2)\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

14 Write an iterated integral for the surface area of the paraboloid

$$z = x^2 + y^2,$$

for $z \leq 4$, parametrised by the cylindrical coordinates r and θ.

a $\int_0^{2\pi} \int_0^4 r\sqrt{4r^2+1}\,\mathrm{d}r\,\mathrm{d}\theta$

b $\int_0^{2\pi} \int_0^2 \sqrt{4r^2+1}\,\mathrm{d}r\,\mathrm{d}\theta$

c $\int_0^{2\pi} \int_0^2 r\sqrt{4r^2+1}\,\mathrm{d}r\,\mathrm{d}\theta$

d $\int_0^{2\pi} \int_0^4 \sqrt{4r^2+1}\,\mathrm{d}r\,\mathrm{d}\theta$

Answers

1 B, 2 D, 3 A, 4 B, 5 A, 6 D, 7 C, 8 B, 9 C, 10 A, 11 D, 12 C, 13 B, 14 C