There will be a comprehensive final exam taken in class on March 16 Wednesday. The exam will be multiple choice, with no partial credit (except possibly on extra credit problems).

For the exam, you may use one sheet of notes that you wrote yourself. However, you may not use your book or anything else not written by you. You certainly should not talk to other people! Calculators are allowed, although you shouldn't really need them, but not communication devices (like cell phones).

1 Given

$$\mathbf{r} = \langle t^2, 3t, 5\rangle = t^2\mathbf{i} + 3t\mathbf{j} + 5\mathbf{k},$$

find the velocity $\mathbf{v} = \mathrm{d}\mathbf{r}/\mathrm{d}t$ and the acceleration $\mathbf{a} = \mathrm{d}\mathbf{v}/\mathrm{d}t$ when $t = 2$.

a $\mathbf{v} = \langle 4, 6, 5\rangle = 4\mathbf{i} + 6\mathbf{j} + 5\mathbf{k},\ \mathbf{a} = \langle 2, 0, 5\rangle = 2\mathbf{i} + 5\mathbf{k}$

b $\mathbf{v} = \langle 4, 3, 0\rangle = 4\mathbf{i} + 3\mathbf{j},\ \mathbf{a} = \langle 2, 0, 0\rangle = 2\mathbf{i}$

c $\mathbf{v} = \langle 4, 3, 0\rangle = 4\mathbf{i} + 3\mathbf{j},\ \mathbf{a} = \langle 2, 0, 5\rangle = 2\mathbf{i} + 5\mathbf{k}$

d $\mathbf{v} = \langle 4, 6, 5\rangle = 4\mathbf{i} + 6\mathbf{j} + 5\mathbf{k},\ \mathbf{a} = \langle 4, 3, 0\rangle = 4\mathbf{i} + 3\mathbf{j}$

2 Given

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \left\langle -3t^6, \frac{7}{t}, 4t^3\right\rangle = -3t^6\mathbf{i} + \frac{7\mathbf{j}}{t} + 4t^3\mathbf{k}$$

for $t > 0$ and $\mathbf{r} = \langle -1, -5, 7\rangle$ when $t = 1$, find $\mathbf{r}$ as a function of t.

a $\mathbf{r} = \left\langle -\frac{3t^7}{7} - 1, 7\ln t - 5, t^4 + 7\right\rangle = \left(-\frac{3t^7}{7} - 1\right)\mathbf{i} + (7\ln t - 5)\mathbf{j} + (t^4 + 7)\mathbf{k}$

b $\mathbf{r} = \left\langle -\frac{3t^7}{7} - \frac{4}{7}, 7\ln t - 5, t^4 + 6\right\rangle = \left(-\frac{3t^7}{7} - \frac{4}{7}\right)\mathbf{i} + (7\ln t - 5)\mathbf{j} + (t^4 + 6)\mathbf{k}$

c $\mathbf{r} = \left\langle -18t^5 + 17, -\frac{7}{t^2} + 2, 12t^2 - 5\right\rangle = (-18t^5 + 17)\mathbf{i} + \left(-\frac{7}{t^2} + 2\right)\mathbf{j} + (12t^2 - 5)\mathbf{k}$

d $\mathbf{r} = \left\langle -18t^5 - 1, -\frac{7}{t^2} - 5, 12t^2 + 7\right\rangle = (-18t^5 - 1)\mathbf{i} + \left(-\frac{7}{t^2} - 5\right)\mathbf{j} + (12t^2 + 7)\mathbf{k}$

3 Find

$$\lim_{(x,y)\to(1,1)} \frac{x^2y - xy^2}{x^2 - y^2}.$$

a undefined (nonexistent or infinite)

b -1

c 0

d $1/2$

4 Find the (first) partial derivatives of

$$u = y^2 \cos(3x)$$

with respect to x and y.

a $\frac{\partial u}{\partial x} = -3\sin(3x),\ \frac{\partial u}{\partial y} = 2y$

b $\frac{\partial u}{\partial x} = -3y^2\sin(3x),\ \frac{\partial u}{\partial y} = 2y\cos(3x)$

c $\frac{\partial u}{\partial x} = -6y\sin(3x),\ \frac{\partial u}{\partial y} = -6y\sin(3x)$

d $\frac{\partial u}{\partial x} = y^2 - 3\sin(3x),\ \frac{\partial u}{\partial y} = 2y + \cos(3x)$

5 Given

$$f(x,y,z) = xy^2 + yz^2 + x^2 z,$$

find the gradient $\nabla f(1,2,3)$.

a $\langle 10, 22, 15\rangle = 10\mathbf{i} + 22\mathbf{j} + 15\mathbf{k}$

b $\langle 10, 13, 13\rangle = 10\mathbf{i} + 13\mathbf{j} + 13\mathbf{k}$

c $\langle 2, 4, 6\rangle = 2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$

d $\langle 6, 13, 7\rangle = 6\mathbf{i} + 13\mathbf{j} + 7\mathbf{k}$

6 Given

$$g(x,y) = x^2 + 2y^2 - 1,$$

find the maximum value of g.

a 0

b none (or ∞)

c 1

d -1

7 If C is the oriented curve given by

$$x = \cos t,\ y = \sin t,\ 0 \le t \le 2\pi$$

(oriented with increasing t), write

$$\int_C (x\,\mathrm{d}x + 2y\,\mathrm{d}x - x\,\mathrm{d}y)$$

as an ordinary integral in t.

a $\int_0^{2\pi} (-\sin t\cos t - 2\sin^2 t - \cos^2 t)\,\mathrm{d}t$

b $\int_0^{2\pi} (-\cos^2 t - 3\sin t\cos t)\,\mathrm{d}t$

c $\int_0^{2\pi} (\cos^2 t + \sin t\cos t)\,\mathrm{d}t$

d $\int_0^{2\pi} (\sin t\cos t + 2\sin^2 t - \cos^2 t)\,\mathrm{d}t$

8 Write the flux of

$$\mathbf{F}(x,y) = \langle x + 2y, -x\rangle = (x+2y)\mathbf{i} - x\mathbf{j}$$

across the circle

$$\{x, y \mid x^2 + y^2 = 1\},$$

in the direction away from the origin, as an ordinary integral in the polar coordinate θ.

a $\int_0^{2\pi} (\cos^2\theta + \sin\theta\cos\theta)\,\mathrm{d}\theta$

b $\int_0^{2\pi} (\sin^2\theta + \sin\theta\cos\theta + 1)\,\mathrm{d}\theta$

c $\int_0^{2\pi} (-\sin^2\theta - \sin\theta\cos\theta - 1)\,\mathrm{d}\theta$

d $\int_0^{2\pi} (-\cos^2\theta - \sin\theta\cos\theta)\,\mathrm{d}\theta$

9 Given

$$(x, y, z) = (5\cos t, 3\sin t, 4\sin t),$$

find the length of this parametrized curve from $t = 0$ to $t = 2\pi$.

a 12π

b 12

c 10

d 10π

10 Write

$$\int_0^1 \int_0^y \mathrm{e}^{-x^2}\, \mathrm{d}x\, \mathrm{d}y$$

as an iterated integral ending in $\mathrm{d}y\,\mathrm{d}x$.

a $\int_0^1 \int_x^1 \mathrm{e}^{-x^2}\, \mathrm{d}y\, \mathrm{d}x$

b $\int_y^1 \int_0^1 \mathrm{e}^{-x^2}\, \mathrm{d}y\, \mathrm{d}x$

c $\int_0^1 \int_0^x \mathrm{e}^{-x^2}\, \mathrm{d}y\, \mathrm{d}x$

d $\int_0^y \int_0^1 \mathrm{e}^{-x^2}\, \mathrm{d}y\, \mathrm{d}x$

11 Write the volume of the pyramid bounded by $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$ as an iterated integral ending in $\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$.

a $\int_0^1 \int_0^1 \int_0^{1-z} \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z$

b $\int_0^1 \int_0^{1-z} \int_0^{1-y-z} \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z$

c $\int_0^1 \int_0^1 \int_0^{1-y-z} \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z$

d $\int_0^1 \int_0^{1-z} \int_0^{1-z} \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z$

12 Write the integral of $3r^2\cos\theta$, on the region above the horizontal axis with a distance between 2 and 5 from the origin, as an iterated integral in polar coordinates.

a $\int_0^{\pi} \int_2^5 3r^3 \cos\theta\, \mathrm{d}r\, \mathrm{d}\theta$

b $\int_0^{2\pi} \int_2^5 3r \cos\theta\, \mathrm{d}r\, \mathrm{d}\theta$

c $\int_0^{2\pi} \int_2^5 3r^3 \cos\theta\, \mathrm{d}r\, \mathrm{d}\theta$

d $\int_0^{\pi} \int_2^5 3r \cos\theta\, \mathrm{d}r\, \mathrm{d}\theta$

13 Write

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{9-x^2-y^2} (x^2+y^2+z^2)\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$$

as an iterated integral in cylindrical coordinates.

a $\int_0^{\pi/2} \int_0^2 \int_0^{9-4r^2} (r^2+z^2)\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

b $\int_0^{\pi/2} \int_0^2 \int_0^{9-r^2} (r^3+rz^2)\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

c $\int_0^{\pi/2} \int_0^2 \int_0^{9-4r^2} (r^3+rz^2)\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

d $\int_0^{\pi/2} \int_0^2 \int_0^{9-r^2} (r^2+z^2)\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

14 Write an integral for the volume of the region above $x^2+y^2=3z^2$ and below $x^2+y^2+z^2=1$ as an iterated integral in spherical coordinates.

a $\int_0^{\pi} \int_0^{\pi} \int_0^1 \rho^2 \sin\phi\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$

b $\int_0^{2\pi} \int_0^{\pi/3} \int_0^1 \rho^2 \sin\phi\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$

c $\int_0^{2\pi} \int_0^{\pi} \int_0^1 \rho \sin\phi\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$

d $\int_0^{\pi} \int_0^{\pi/3} \int_0^1 \rho \sin\phi\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$

15 Write an iterated integral for the surface area of the cone

$$z^2 = x^2 + y^2,$$

for $0 \le z \le 4$, parametrized by the cylindrical coordinates r and θ.

a $\int_0^{2\pi} \int_0^4 \sqrt{2r^2+1}\,\mathrm{d}r\,\mathrm{d}\theta$

b $\int_0^{2\pi} \int_0^4 \sqrt{2r+1}\,\mathrm{d}r\,\mathrm{d}\theta$

c $\int_0^{2\pi} \int_0^4 \sqrt{2r}\,\mathrm{d}r\,\mathrm{d}\theta$

d $\int_0^{2\pi} \int_0^4 \sqrt{2}\,r\,\mathrm{d}r\,\mathrm{d}\theta$

16 Find the curl and divergence of

$$\mathbf{F}(x,y) = \langle 2xy, -3xy \rangle = 2xy\mathbf{i} - 3xy\mathbf{j}.$$

a $\nabla \times \mathbf{F}(x,y) = 2x-3y,\ \nabla \cdot \mathbf{F}(x,y) = -3x-2y$

b $\nabla \times \mathbf{F}(x,y) = -2x-3y,\ \nabla \cdot \mathbf{F}(x,y) = -3x+2y$

c $\nabla \times \mathbf{F}(x,y) = -3x+2y,\ \nabla \cdot \mathbf{F}(x,y) = 2x-3y$

d $\nabla \times \mathbf{F}(x,y) = -3x-2y,\ \nabla \cdot \mathbf{F}(x,y) = -2x-3y$

17 If C is the counterclockwise-oriented curve given by

$$x = \cos t,\ y = \sin t,\ 0 \le t \le 2\pi,$$

use Green's Theorem to write

$$\int_C (x\,\mathrm{d}x + 2y\,\mathrm{d}x - x\,\mathrm{d}y)$$

as a double integral (with respect to area) over the region R where $x^2 + y^2 \le 1$.

a $\iint_R (-3)\,\mathrm{d}A$

b $\iint_R (-3)xy\,\mathrm{d}A$

c $\iint_R 3xy\,\mathrm{d}A$

d $\iint_R 3\,\mathrm{d}A$

Answers

1 B, 2 B, 3 D, 4 B, 5 B, 6 B, 7 A, 8 A, 9 D, 10 A, 11 B, 12 A, 13 B, 14 B, 15 D, 16 B, 17 A