

The Final Exam consists of questions similar in style and content to those below. (There are answers at the end of this practice exam.)

1 Evaluate

$$\lim_{x \rightarrow -3^-} \left(\frac{x^2 + 3}{x + 3} \right).$$

- a.* 6
- b.* undefined
- c.* $-\infty$
- d.* ∞

2 Evaluate

$$\lim_{x \rightarrow -\infty} \frac{e^x + 4x}{3x}.$$

- a.* $\frac{4}{3}$
- b.* 0
- c.* ∞
- d.* $\frac{1}{3}$

3 Evaluate

$$\lim_{x \rightarrow -\infty} (x^4 + 5x^2).$$

- a.* 24
- b.* ∞
- c.* $-\infty$
- d.* undefined

4 Evaluate

$$\lim_{x \rightarrow -2} \left(\frac{x^2 - 5x - 14}{x^2 + 5x + 6} \right).$$

- a.* $-\infty$
- b.* -9
- c.* ∞
- d.* 0

5 Evaluate

$$\lim_{x \rightarrow 0} \left(\frac{8 - 8e^{-5x} + 2x}{\cos x + x - 1} \right).$$

- a.* $-\infty$
- b.* ∞
- c.* 42
- d.* 0

6 Given

$$f(x) = x^2 - 3x,$$

use the definition of the derivative as a limit to calculate $f'(1)$.

a. $\lim_{h \rightarrow 0} (h^2 - 3h - 1) = -1$

b. $\lim_{h \rightarrow 0} (h - 1) = -1$

c. $\lim_{h \rightarrow 0} (2h - 1) = -1$

d. $\lim_{h \rightarrow 0} \left(\frac{2h - 3}{3} \right) = -1$

7 Given

$$y = x^{2/7} - 2e^x,$$

find the derivative of y with respect to x .

a. $\frac{dy}{dx} = \frac{2}{7}x^{9/7} - 2xe^{x-1}$

b. $\frac{dy}{dx} = \frac{2}{7}x^{-5/7} - 2xe^{x-1}$

c. $\frac{dy}{dx} = \frac{2}{7}x^{9/7} - 2e^x$

d. $\frac{dy}{dx} = \frac{2}{7}x^{-5/7} - 2e^x$

8 Given

$$xy^3 - 4y^2 = x^2,$$

find the derivative of y with respect to x .

a. $\frac{dy}{dx} = \frac{2x - y^3}{3xy - 8}$

b. $\frac{dy}{dx} = \frac{2x - y^3}{3xy^2 - 8y}$

c. $\frac{dy}{dx} = \frac{1}{3y - 8}$

d. $\frac{dy}{dx} = \frac{1}{3y^2 - 8y}$

9 Given

$$x = t^2 \sin(e^t),$$

find the derivative of x with respect to t .

a. $\frac{dx}{dt} = 2t \sin(e^t) + t^2 \cos(e^t)$

b. $\frac{dx}{dt} = 2t \cos(e^t)$

c. $\frac{dx}{dt} = 2te^t \cos(e^t)$

d. $\frac{dx}{dt} = 2t \sin(e^t) + t^2 e^t \cos(e^t)$

10 Given

$$p = \arctan(q^2) + \cos(3q) = \tan^{-1}(q^2) + \cos(3q),$$

find the derivative of p with respect to q .

a. $\frac{2q}{\sqrt{1-q^4}} + 3\sin(3q)$

b. $\frac{2q}{q^4+1} - 3\sin(3q)$

c. $\frac{2q}{\sqrt{1-q^4}} - 3\sin(3q)$

d. $\frac{2q}{q^4+1} + 3\sin(3q)$

11 Given

$$f(x) = \frac{x+1}{x-4},$$

find f' .

a. $f'(x) = -\frac{5}{(x+1)^2}$

b. $f'(x) = -\frac{5}{(x-4)^2}$

c. $f'(x) = \frac{5}{(x-4)^2}$

d. $f'(x) = \frac{5}{(x+1)^2}$

12 Given

$$g(x) = 3\ln(x^2+1),$$

find g'' .

a. $g''(x) = \frac{6-6x^2}{(x^2+1)^2}$

b. $g''(x) = \frac{6x}{x^2+1}$

c. $g''(x) = 6x\ln(x^2+1)$

d. $g''(x) = \frac{6+18x^2}{(x^2+1)^2}$

13 Given

$$f(x) = \sqrt{4-x^2},$$

find the maximum and minimum value of f , if they exist.

a. no maximum, minimum is 0

b. maximum is 2, no minimum

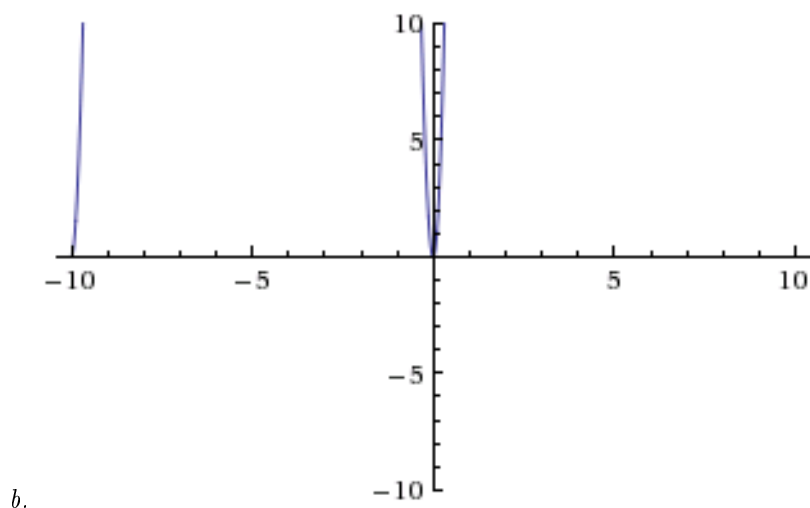
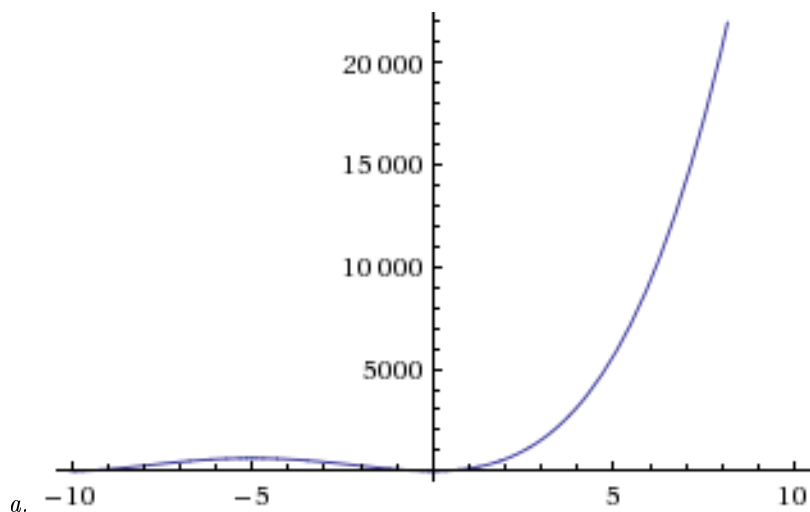
c. no maximum, no minimum

d. maximum is 2, minimum is 0

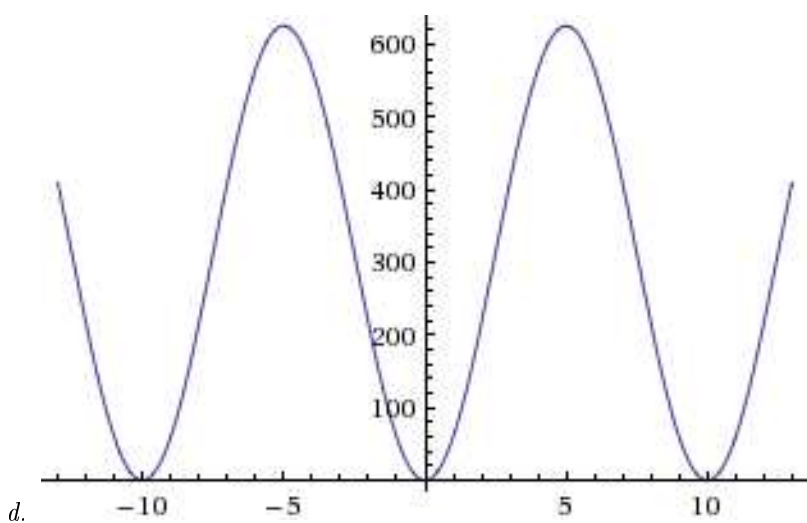
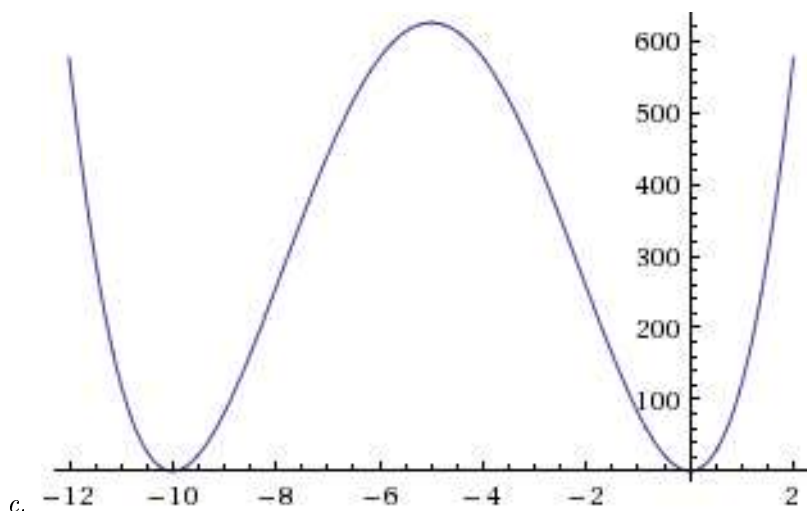
14 Given

$$f(x) = x^4 + 20x^3 + 100x^2,$$

sketch a graph of f that shows all intercepts (if any), all turning points (if any), and all linear asymptotes (if any).



(More answers on the next page)



15 Given

$$f(x) = 2e^x - \frac{10}{x} + 8x^3,$$

find the antiderivatives (indefinite integrals) of f .

a. $\int f(x) \, dx = 2e^x - 10 \ln |x| + 24x^2 + C$

b. $\int f(x) \, dx = 2e^x - \frac{10}{x^2} + 2x^4 + C$

c. $\int f(x) \, dx = 2e^x - 10 \ln |x| + 2x^4 + C$

d. $\int f(x) \, dx = 2e^x - \frac{10}{x^2} + 24x^2 + C$

16 Find the value of

$$\int_0^{\pi/3} 4 \sec x \tan x \, dx.$$

a. 0

b. 4

c. 2

d. 3

17 Find the value of

$$\int_0^{\pi/2} \frac{\cos x}{\sqrt{4 + 3 \sin x}} dx.$$

a. $\frac{2}{3}\sqrt{7} - \frac{4}{3} \approx 0.43$

b. 0

c. $6\sqrt{7} - 12 \approx 3.87$

d. $2\sqrt{7} - 4 \approx 1.29$

18 Set up a definite integral whose value is the area bounded by the graphs of these equations:

$$y = x^2,$$
$$y = x + 6.$$

a. $\int_{-2}^3 (x^2 - x - 6) dx$

b. $\int_4^9 (x^2 - x - 6) dx$

c. $\int_{-2}^3 (x + 6 - x^2) dx$

d. $\int_4^9 (x + 6 - x^2) dx$

19 Set up an integral whose value is the length of the curve with equation

$$y = 2 - \frac{x^2}{4}$$

from $(x, y) = (0, 2)$ to $(x, y) = (2, 1)$.

a. $\int_1^2 \frac{1}{2} \sqrt{4 + x^2} dx$

b. $\int_2^0 \frac{1}{2} \sqrt{4 + x^2} dx$

c. $\int_2^1 \frac{1}{2} \sqrt{4 + x^2} dx$

d. $\int_0^2 \frac{1}{2} \sqrt{4 + x^2} dx$

20 Suppose a block of ice is in the form of a cube and is melting. Suppose further that the length of its sides is decreasing by 1/2 inch per hour. Determine the rate of decrease in the volume of the block of ice when the length of a side is 12 inches.

a. 9 cubic inches per hour

b. 216 cubic inches per hour

c. 864 cubic inches per hour

d. 15 cubic inches per hour

- 21** Suppose that a ball thrown into the air has its height given by

$$h = 6 + 5t - 16t^2,$$

where h is its height in feet and t is the time in seconds since it was thrown, from the time it is thrown until the time it lands. What is the ball's maximum height?

- a.* 409/64 ft
- b.* 6 ft
- c.* 5/32 ft
- d.* 5 ft

- 22** Fill in the blank: If

$$\lim_{x \rightarrow c} f(x) = f(c),$$

then f is _____ at c .

- 23** If

$$\lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h}$$

exists, then f is _____ at c .

- 24** The _____ of y is dy .

- 25** The _____ of f is f' .

- 26** The expression $\int_a^b f(x) \, dx$ is a(n) _____ integral.

- 27** The expression $\int f(x) \, dx$ is a(n) _____ integral.

Answers

1 C, 2 A, 3 B, 4 B, 5 C, 6 B, 7 D, 8 B, 9 D, 10 B, 11 B, 12 A, 13 D, 14 C, 15 C, 16 B, 17 A, 18 C, 19 D, 20 B, 21 A.

22 continuous

23 differentiable

24 differential

25 derivative

26 definite

27 indefinite