The Final Exam consists of questions similar in style and content to those below. (There are answers at the end of this practice exam.)

1 Given
$$\mathbf{r} = \langle t^2, 3t, 5\rangle = t^2\mathbf{i} + 3t\mathbf{j} + 5\mathbf{k},$$
find the velocity $\mathbf{v} = \mathrm{d}\mathbf{r}/\mathrm{d}t$ and the acceleration $\mathbf{a} = \mathrm{d}\mathbf{v}/\mathrm{d}t$ when $t = 2$.

a. $\mathbf{v} = \langle 4, 6, 5\rangle = 4\mathbf{i} + 6\mathbf{j} + 5\mathbf{k},\ \mathbf{a} = \langle 2, 0, 5\rangle = 2\mathbf{i} + 5\mathbf{k}$

b. $\mathbf{v} = \langle 4, 3, 0\rangle = 4\mathbf{i} + 3\mathbf{j},\ \mathbf{a} = \langle 2, 0, 0\rangle = 2\mathbf{i}$

c. $\mathbf{v} = \langle 4, 3, 0\rangle = 4\mathbf{i} + 3\mathbf{j},\ \mathbf{a} = \langle 2, 0, 5\rangle = 2\mathbf{i} + 5\mathbf{k}$

d. $\mathbf{v} = \langle 4, 6, 5\rangle = 4\mathbf{i} + 6\mathbf{j} + 5\mathbf{k},\ \mathbf{a} = \langle 4, 3, 0\rangle = 4\mathbf{i} + 3\mathbf{j}$

2 Given
$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \left\langle -3t^6, \frac{7}{t}, 4t^3\right\rangle = -3t^6\mathbf{i} + \frac{7\mathbf{j}}{t} + 4t^3\mathbf{k}$$
for $t > 0$ and $\mathbf{r} = \langle -1, -5, 7\rangle$ when $t = 1$, find $\mathbf{r}$ as a function of t.

a. $\mathbf{r} = \left\langle -\frac{3t^7}{7} - 1, 7\ln t - 5, t^4 + 7\right\rangle = \left(-\frac{3t^7}{7} - 1\right)\mathbf{i} + (7\ln t - 5)\mathbf{j} + (t^4 + 7)\mathbf{k}$

b. $\mathbf{r} = \left\langle -\frac{3t^7}{7} - \frac{4}{7}, 7\ln t - 5, t^4 + 6\right\rangle = \left(-\frac{3t^7}{7} - \frac{4}{7}\right)\mathbf{i} + (7\ln t - 5)\mathbf{j} + (t^4 + 6)\mathbf{k}$

c. $\mathbf{r} = \left\langle -18t^5 + 17, -\frac{7}{t^2} + 2, 12t^2 - 5\right\rangle = (-18t^5 + 17)\mathbf{i} + \left(-\frac{7}{t^2} + 2\right)\mathbf{j} + (12t^2 - 5)\mathbf{k}$

d. $\mathbf{r} = \left\langle -18t^5 - 1, -\frac{7}{t^2} - 5, 12t^2 + 7\right\rangle = (-18t^5 - 1)\mathbf{i} + \left(-\frac{7}{t^2} - 5\right)\mathbf{j} + (12t^2 + 7)\mathbf{k}$

3 Given
$$(x, y, z) = (5\cos t, 3\sin t, 4\sin t),$$
find the length of this parametrized curve from $t = 0$ to $t = 2\pi$.

a. 12π

b. 12

c. 10

d. 10π

4 Find
$$\lim_{(x,y)\to(1,1)} \frac{x^2y - xy^2}{x^2 - y^2}.$$

a. undefined (inconsistent or infinite)

b. -1

c. 0

d. $1/2$

5 Find the (first) partial derivatives of

$$u = y^2 \cos(3x)$$

with respect to x and y.

a. $\dfrac{\partial u}{\partial x} = -3\sin(3x),\ \dfrac{\partial u}{\partial y} = 2y$

b. $\dfrac{\partial u}{\partial x} = -3y^2\sin(3x),\ \dfrac{\partial u}{\partial y} = 2y\cos(3x)$

c. $\dfrac{\partial u}{\partial x} = -6y\sin(3x),\ \dfrac{\partial u}{\partial y} = -6y\sin(3x)$

d. $\dfrac{\partial u}{\partial x} = y^2 - 3\sin(3x),\ \dfrac{\partial u}{\partial y} = 2y + \cos(3x)$

6 Given

$$f(x,y,z) = xy^2 + yz^2 + x^2 z,$$

find the gradient $\nabla f(1,2,3)$.

a. $\langle 10, 22, 15\rangle = 10\mathbf{i} + 22\mathbf{j} + 15\mathbf{k}$

b. $\langle 10, 13, 13\rangle = 10\mathbf{i} + 13\mathbf{j} + 13\mathbf{k}$

c. $\langle 2, 4, 6\rangle = 2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$

d. $\langle 6, 13, 7\rangle = 6\mathbf{i} + 13\mathbf{j} + 7\mathbf{k}$

7 Given

$$g(x,y) = x^2 + 2y^2 - 1,$$

find the minimum value of g.

a. 0

b. none (unachieved or infinite)

c. 1

d. -1

8 If C is the oriented curve given by

$$x = \cos t,\ y = \sin t,\ 0 \le t \le 2\pi$$

(oriented with increasing t), write

$$\int_C (x\,\mathrm{d}x + 2y\,\mathrm{d}x - x\,\mathrm{d}y)$$

as an ordinary integral in t.

a. $\displaystyle\int_0^{2\pi} (-\sin t\cos t - 2\sin^2 t - \cos^2 t)\,\mathrm{d}t$

b. $\displaystyle\int_0^{2\pi} (-\cos^2 t - 3\sin t\cos t)\,\mathrm{d}t$

c. $\displaystyle\int_0^{2\pi} (\cos^2 t + \sin t\cos t)\,\mathrm{d}t$

d. $\displaystyle\int_0^{2\pi} (\sin t\cos t + 2\sin^2 t - \cos^2 t)\,\mathrm{d}t$

9 Write the flux of

$$\mathbf{F}(x,y) = \langle x+2y, -x\rangle = (x+2y)\mathbf{i} - x\mathbf{j}$$

across the circle

$$\{x, y \mid x^2 + y^2 = 1\},$$

in the direction away from the origin, as an ordinary integral in the polar coordinate θ.

a. $\displaystyle\int_0^{2\pi} (\cos^2\theta + \sin\theta\cos\theta)\,\mathrm{d}\theta$

b. $\displaystyle\int_0^{2\pi} (\sin^2\theta + \sin\theta\cos\theta + 1)\,\mathrm{d}\theta$

c. $\displaystyle\int_0^{2\pi} (-\sin^2\theta - \sin\theta\cos\theta - 1)\,\mathrm{d}\theta$

d. $\displaystyle\int_0^{2\pi} (-\cos^2\theta - \sin\theta\cos\theta)\,\mathrm{d}\theta$

10 Write

$$\int_0^1 \int_0^y \mathrm{e}^{-x^2}\,\mathrm{d}x\,\mathrm{d}y$$

as an iterated integral ending in $\mathrm{d}y\,\mathrm{d}x$.

a. $\displaystyle\int_0^1 \int_x^1 \mathrm{e}^{-x^2}\,\mathrm{d}y\,\mathrm{d}x$

b. $\displaystyle\int_y^1 \int_0^1 \mathrm{e}^{-x^2}\,\mathrm{d}y\,\mathrm{d}x$

c. $\displaystyle\int_0^1 \int_0^x \mathrm{e}^{-x^2}\,\mathrm{d}y\,\mathrm{d}x$

d. $\displaystyle\int_0^y \int_0^1 \mathrm{e}^{-x^2}\,\mathrm{d}y\,\mathrm{d}x$

11 Write the volume of the pyramid bounded by $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$ as an iterated integral ending in $\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$.

a. $\displaystyle\int_0^1 \int_0^1 \int_0^{1-z} \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

b. $\displaystyle\int_0^1 \int_0^{1-z} \int_0^{1-y-z} \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

c. $\displaystyle\int_0^1 \int_0^1 \int_0^{1-y-z} \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

d. $\displaystyle\int_0^1 \int_0^{1-z} \int_0^{1-z} \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

12 Write the integral of $3r^2\cos\theta$, on the region above the horizontal axis with a distance between 2 and 5 from the origin, as an iterated integral in polar coordinates.

a. $\displaystyle\int_0^{\pi} \int_2^5 3r^3\cos\theta\,\mathrm{d}r\,\mathrm{d}\theta$

b. $\displaystyle\int_0^{2\pi} \int_2^5 3r\cos\theta\,\mathrm{d}r\,\mathrm{d}\theta$

c. $\displaystyle\int_0^{2\pi} \int_2^5 3r^3\cos\theta\,\mathrm{d}r\,\mathrm{d}\theta$

d. $\displaystyle\int_0^{\pi} \int_2^5 3r\cos\theta\,\mathrm{d}r\,\mathrm{d}\theta$

13 Write

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{9-x^2-y^2} (x^2+y^2+z^2)\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$$

as an iterated integral in cylindrical coordinates.

a. $\int_0^{\pi/2} \int_0^2 \int_0^{9-4r^2} (r^2+z^2)\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

b. $\int_0^{\pi/2} \int_0^2 \int_0^{9-r^2} (r^3+rz^2)\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

c. $\int_0^{\pi/2} \int_0^2 \int_0^{9-4r^2} (r^3+rz^2)\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

d. $\int_0^{\pi/2} \int_0^2 \int_0^{9-r^2} (r^2+z^2)\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

14 Write an integral for the volume of the region above $x^2+y^2=3z^2$ and below $x^2+y^2+z^2=1$ as an iterated integral in spherical coordinates.

a. $\int_0^{\pi} \int_0^{\pi} \int_0^1 \rho^2 \sin\varphi\,\mathrm{d}\rho\,\mathrm{d}\varphi\,\mathrm{d}\theta$

b. $\int_0^{2\pi} \int_0^{\pi/3} \int_0^1 \rho^2 \sin\varphi\,\mathrm{d}\rho\,\mathrm{d}\varphi\,\mathrm{d}\theta$

c. $\int_0^{2\pi} \int_0^{\pi} \int_0^1 \rho \sin\varphi\,\mathrm{d}\rho\,\mathrm{d}\varphi\,\mathrm{d}\theta$

d. $\int_0^{\pi} \int_0^{\pi/3} \int_0^1 \rho \sin\varphi\,\mathrm{d}\rho\,\mathrm{d}\varphi\,\mathrm{d}\theta$

15 Write an iterated integral for the surface area of the cone

$$z^2 = x^2 + y^2,$$

for $0 \le z \le 4$, parametrized by the cylindrical coordinates r and θ.

a. $\int_0^{2\pi} \int_0^4 \sqrt{2r^2+1}\,\mathrm{d}r\,\mathrm{d}\theta$

b. $\int_0^{2\pi} \int_0^4 \sqrt{2r+1}\,\mathrm{d}r\,\mathrm{d}\theta$

c. $\int_0^{2\pi} \int_0^4 \sqrt{2r}\,\mathrm{d}r\,\mathrm{d}\theta$

d. $\int_0^{2\pi} \int_0^4 \sqrt{2}\,r\,\mathrm{d}r\,\mathrm{d}\theta$

16 If C is the counterclockwise-oriented curve given by

$$x = \cos t,\ y = \sin t,\ 0 \le t \le 2\pi,$$

use Green's Theorem to write

$$\int_C (x\,\mathrm{d}x + 2y\,\mathrm{d}x - x\,\mathrm{d}y)$$

as a double integral (with respect to area) over the region R where $x^2+y^2 \le 1$.

a. $\iint_R (-3)\,\mathrm{d}A$

b. $\iint_R (-3)xy\,\mathrm{d}A$

c. $\iint_R 3xy\,\mathrm{d}A$

d. $\iint_R 3\,\mathrm{d}A$

17 Find the curl and divergence of

$$\mathbf{F}(x,y) = \langle 2xy, -3xy\rangle = 2xy\mathbf{i} - 3xy\mathbf{j}.$$

a. $\nabla \times \mathbf{F}(x,y) = 2x - 3y,\ \nabla \cdot \mathbf{F}(x,y) = -3x - 2y$

b. $\nabla \times \mathbf{F}(x,y) = -2x - 3y,\ \nabla \cdot \mathbf{F}(x,y) = -3x + 2y$

c. $\nabla \times \mathbf{F}(x,y) = -3x + 2y,\ \nabla \cdot \mathbf{F}(x,y) = 2x - 3y$

d. $\nabla \times \mathbf{F}(x,y) = -3x - 2y,\ \nabla \cdot \mathbf{F}(x,y) = -2x - 3y$

18 Fill in the blank: A function that takes a number as input and gives a point (or a position vector) as output is a(n) __________________ (geometrically).

19 A function that takes a point (with n coordinates) as input and gives a vector (with n components) as output is a(n) __________________ (in n dimensions).

20 If $u = f(x,y)$ (where f is a differentiable function of 2 variables), then $\mathrm{d}u = \mathrm{D}_1 f(x,y)\,\mathrm{d}x + \mathrm{D}_2 f(x,y)\,\mathrm{d}y = \left(\frac{\partial u}{\partial x}\right)_y \mathrm{d}x + \left(\frac{\partial u}{\partial y}\right)_x \mathrm{d}y$ is the __________________ of u.

21 A vector field $\mathbf{F}$ is __________________ if and only if there is a differentiable scalar field f such that $\mathbf{F}$ is the gradient of f.

Answers

1 B, 2 B, 3 D, 4 D, 5 B, 6 B, 7 D, 8 A, 9 A, 10 A, 11 B, 12 A, 13 B, 14 B, 15 D, 16 A, 17 B

18 parametrized curve

19 vector field

20 differential

21 conservative